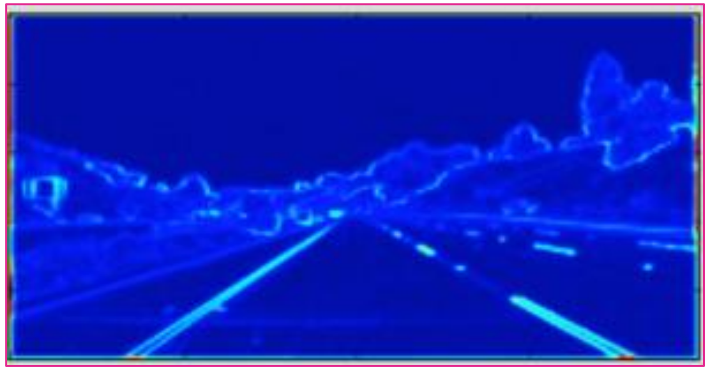
Edge:
Hough transform

Dr. Tushar Sandhan

Introduction



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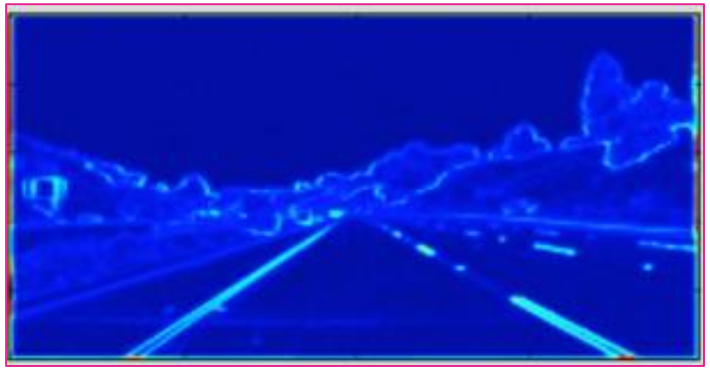

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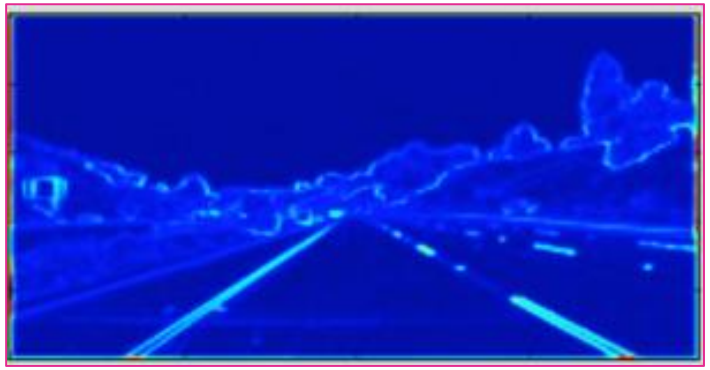

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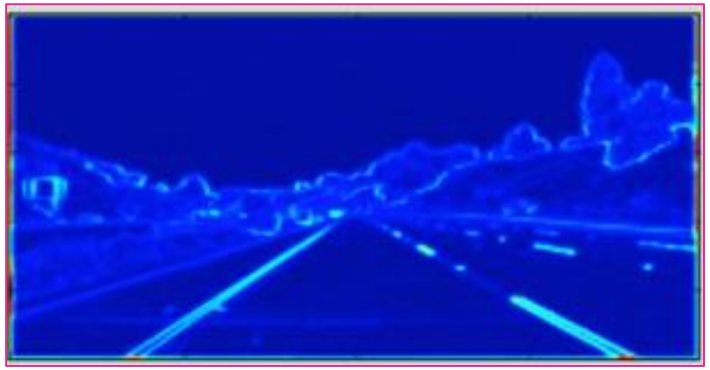

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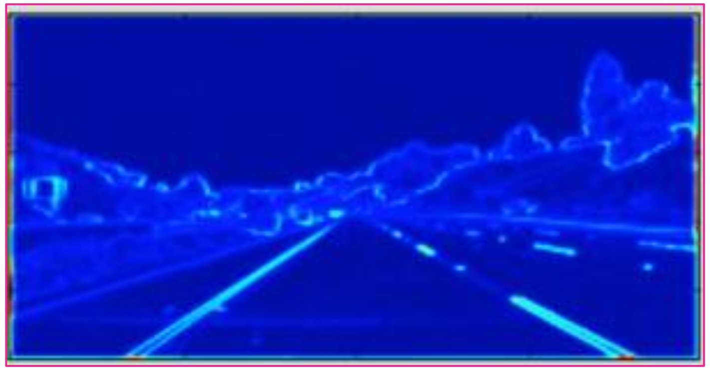

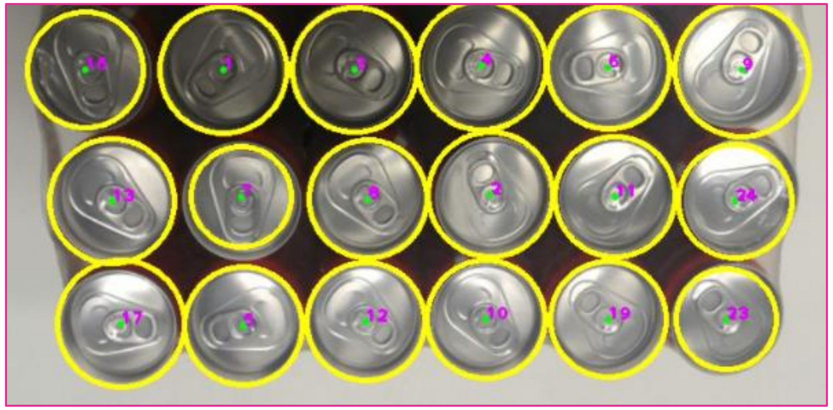

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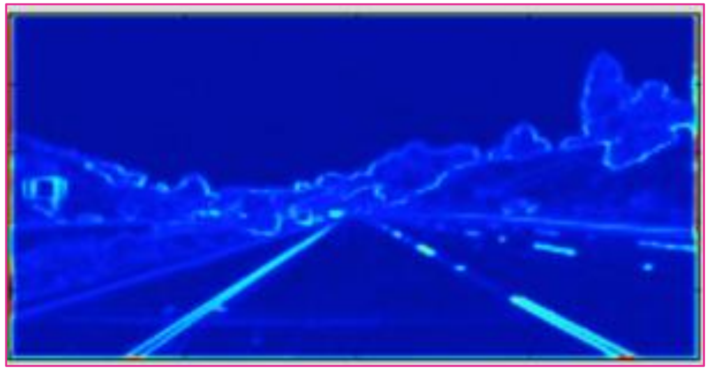

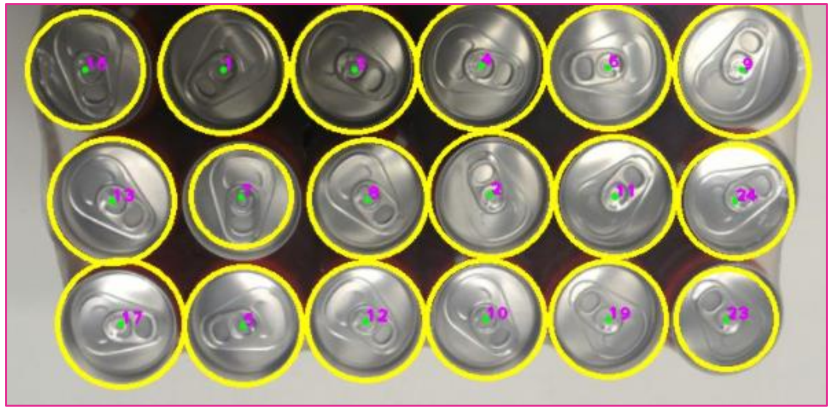

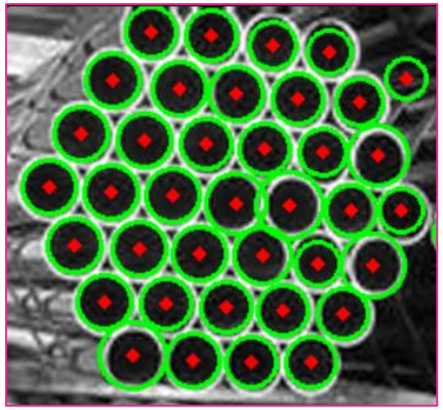

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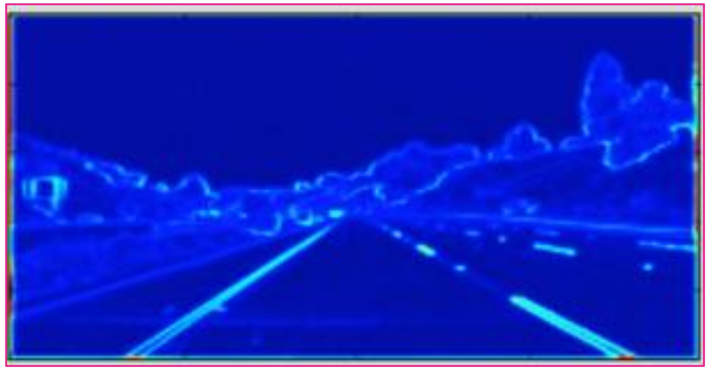

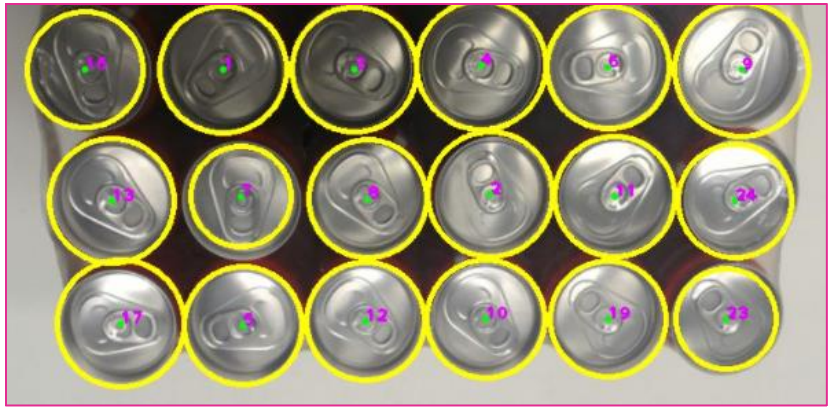

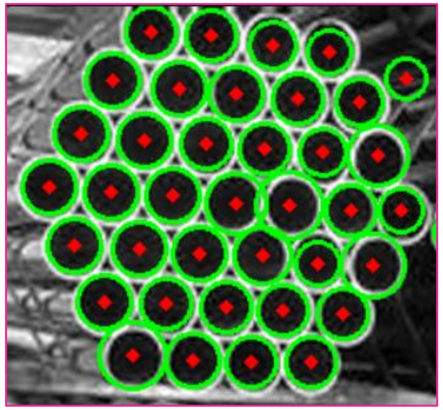

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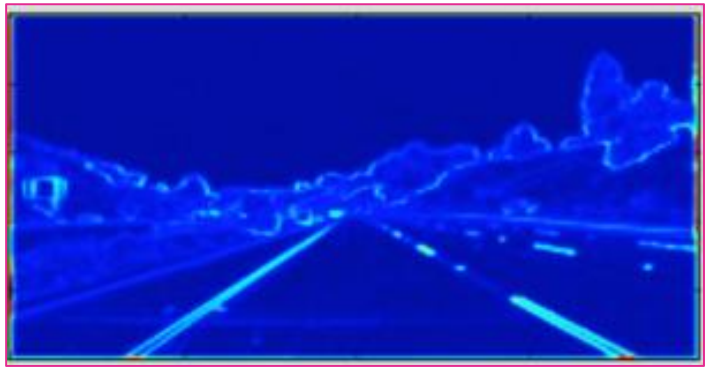

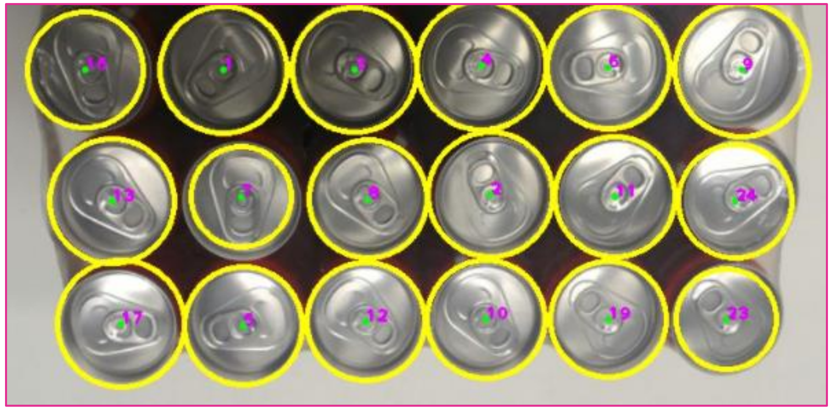

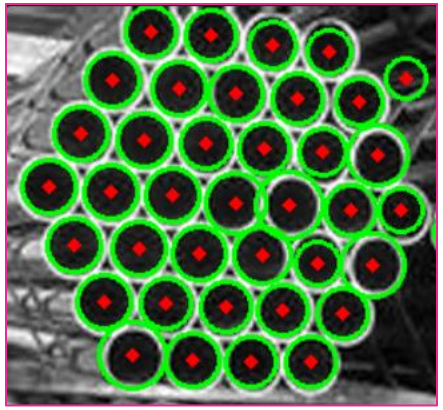

Hough transform

- Image derivatives
  - input image $f(x, y)$
  - (optional) smoothed $f_s(x, y)$
  - get the gradients $g_x(x, y)$, $g_y(x, y)$
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  - considers shape of the object as prior info.
  - shape is defined as a function and parametrized

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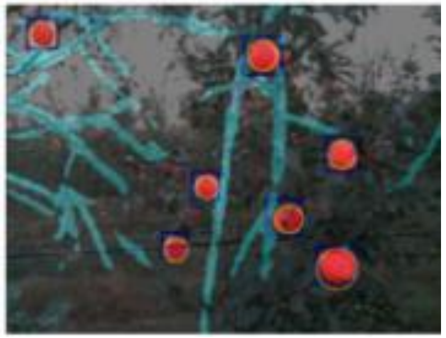

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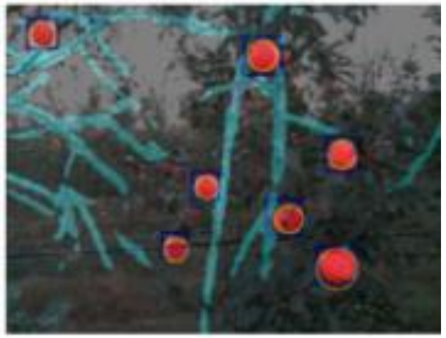

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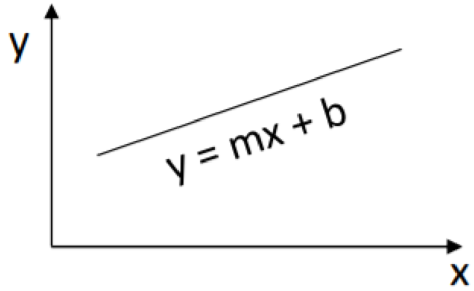

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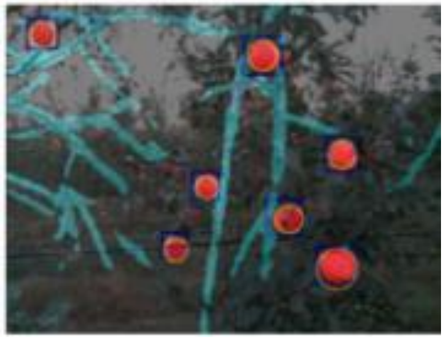

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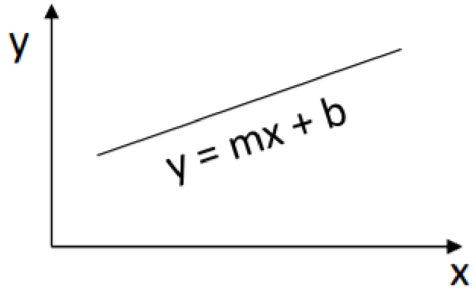

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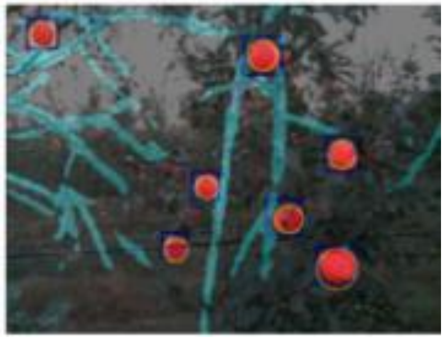

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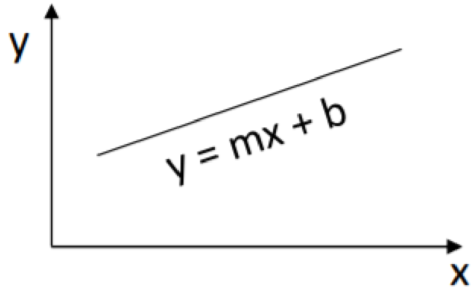

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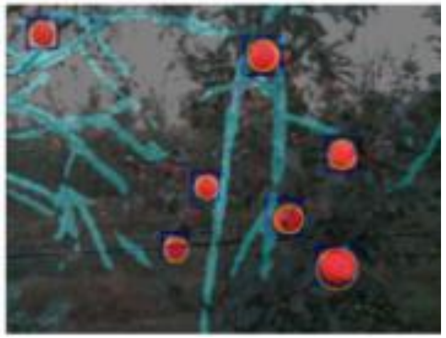

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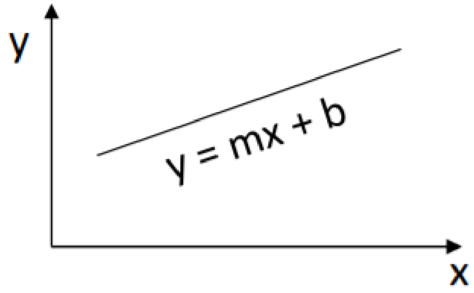

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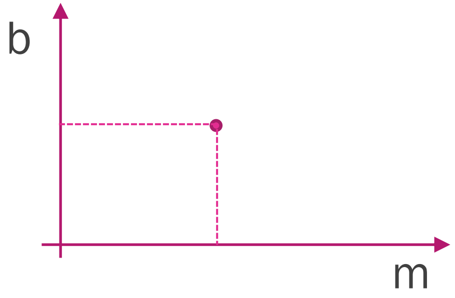

Hough transform

- Hough transform duality
  - Lines in the image space becomes a point in the Hough space
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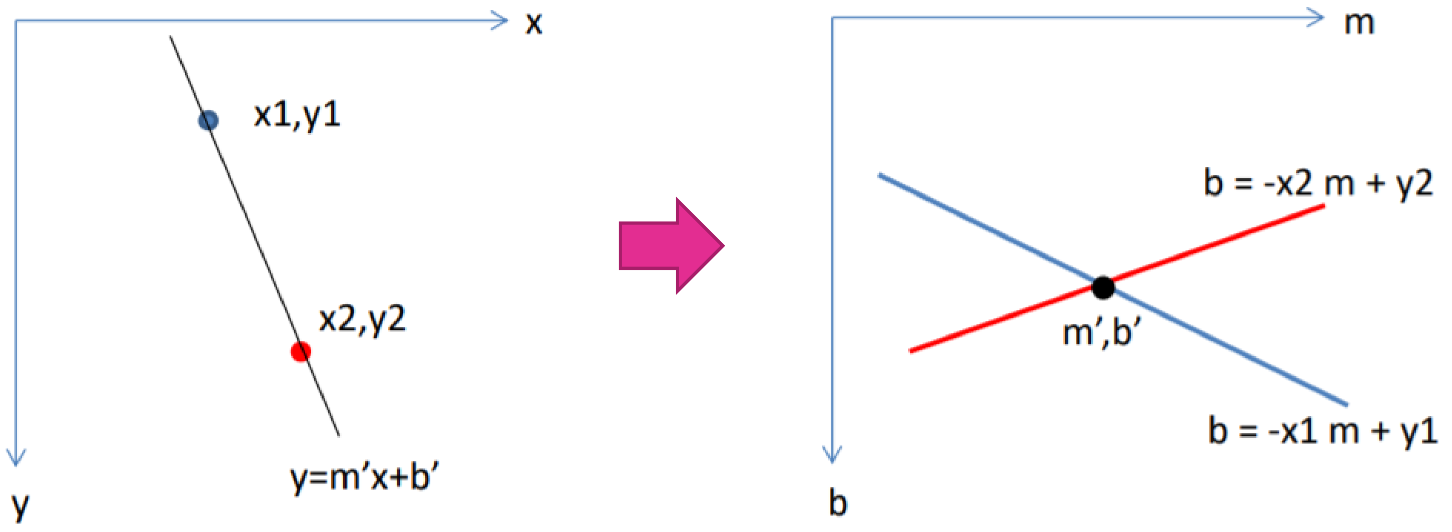

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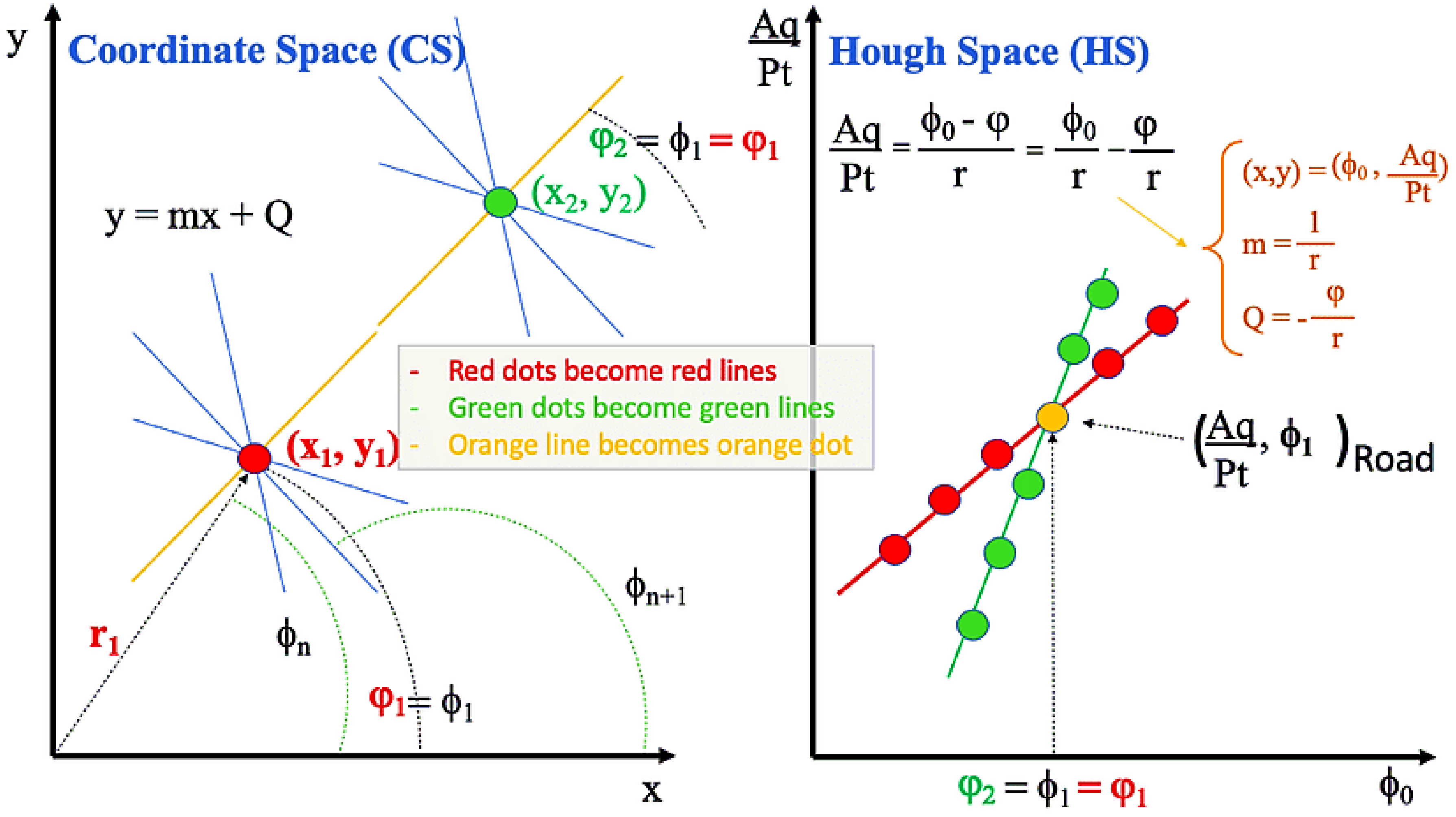

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courtesy: W. Hoff

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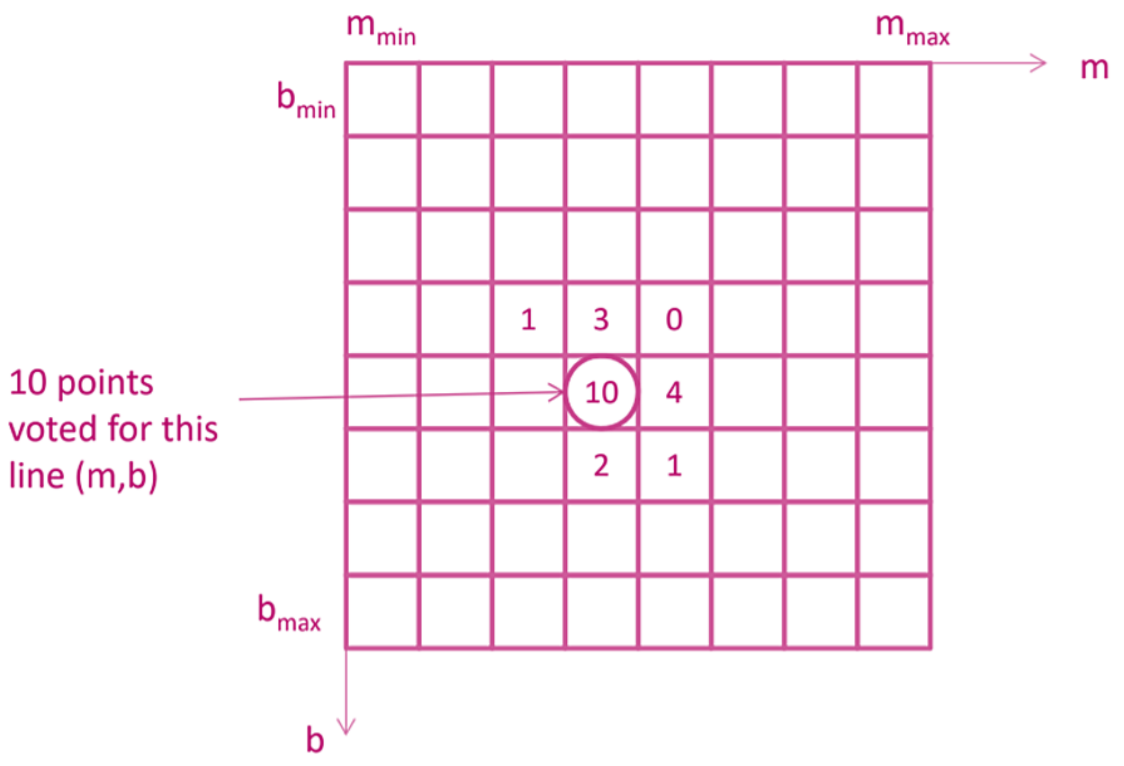

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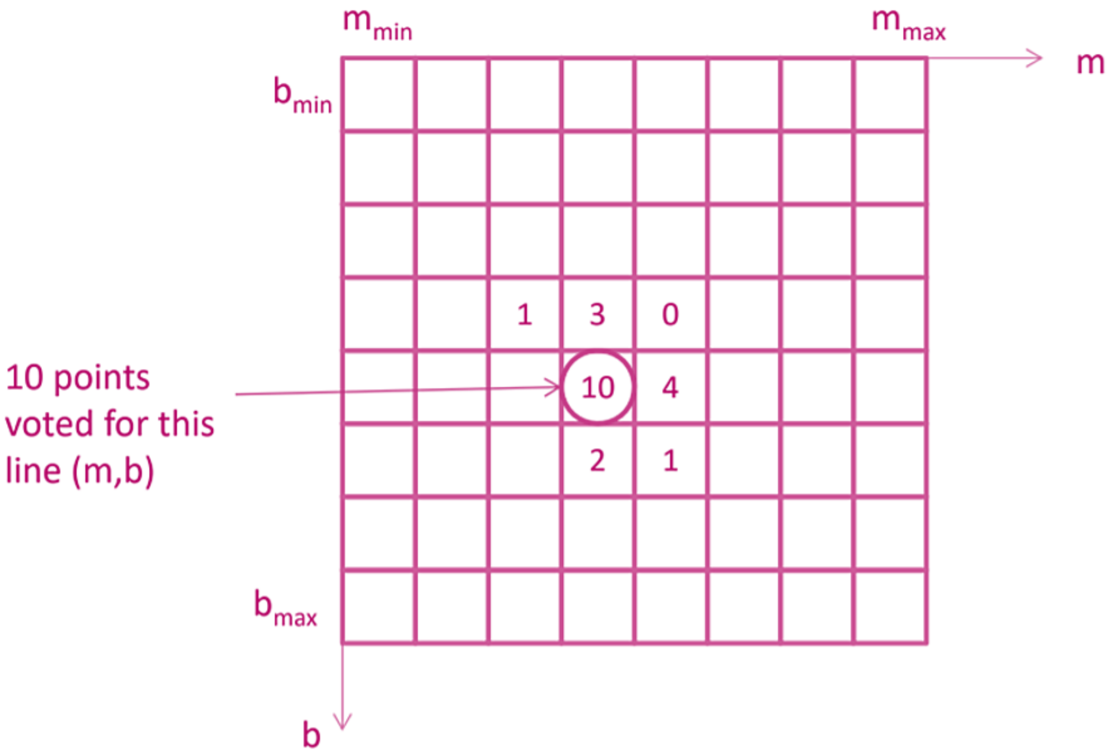

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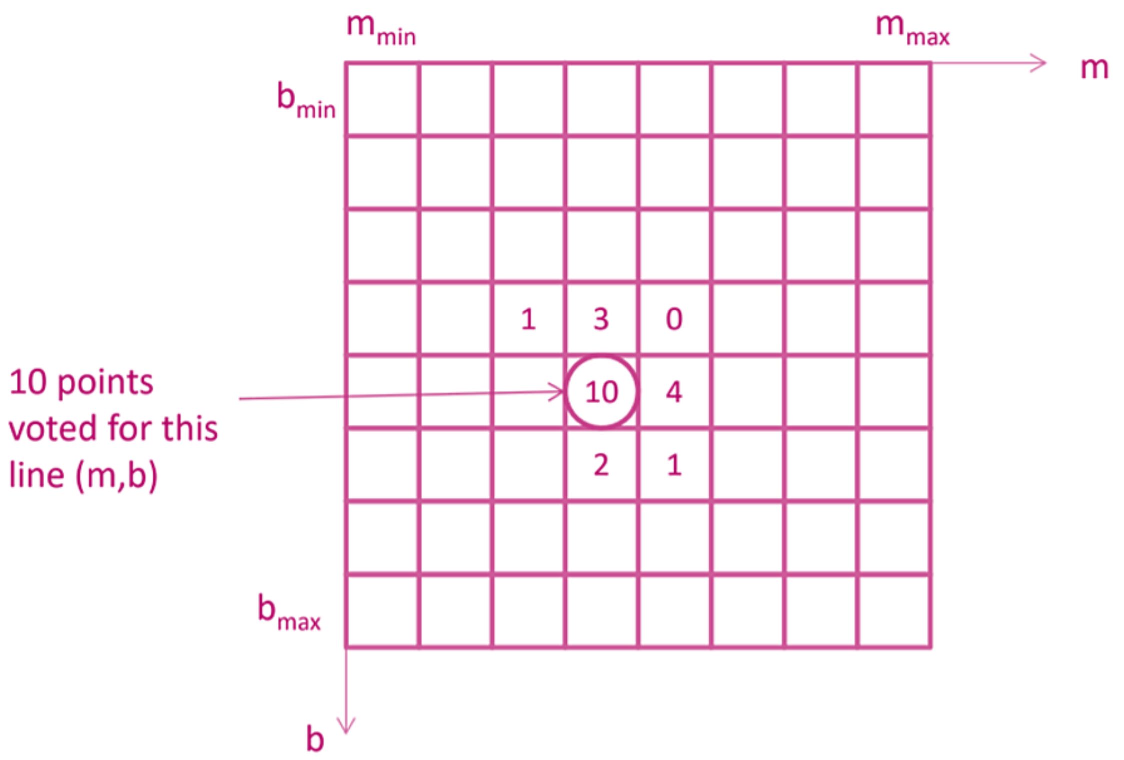

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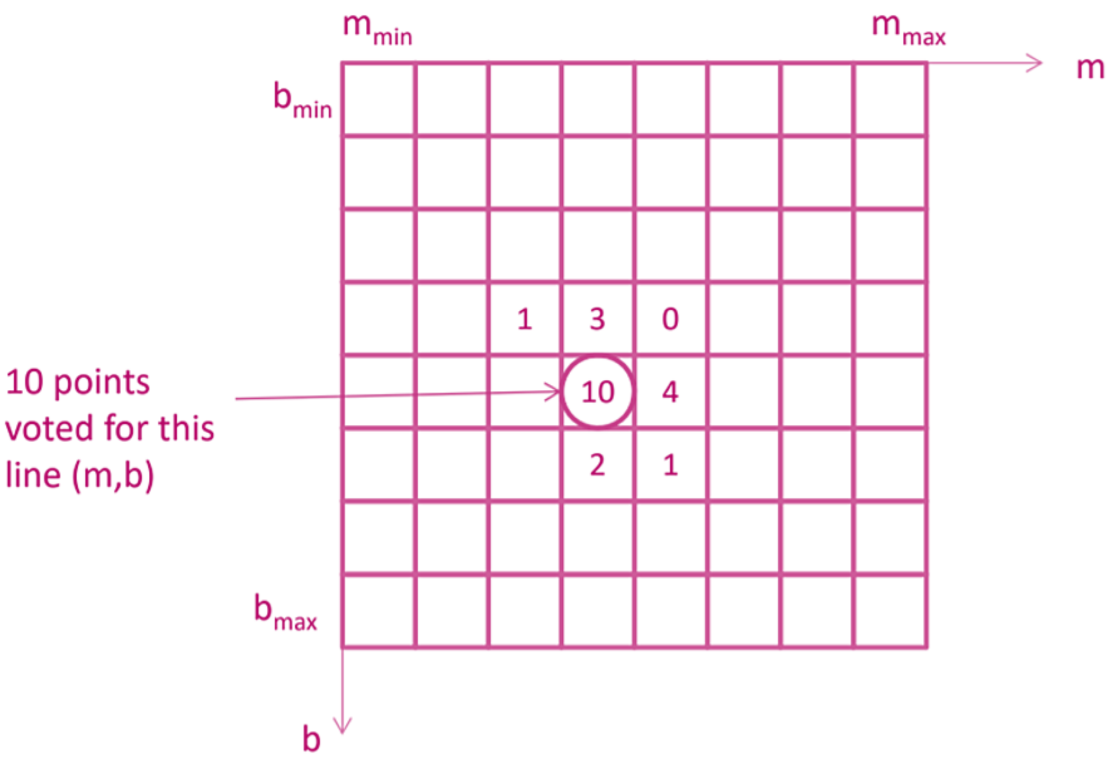

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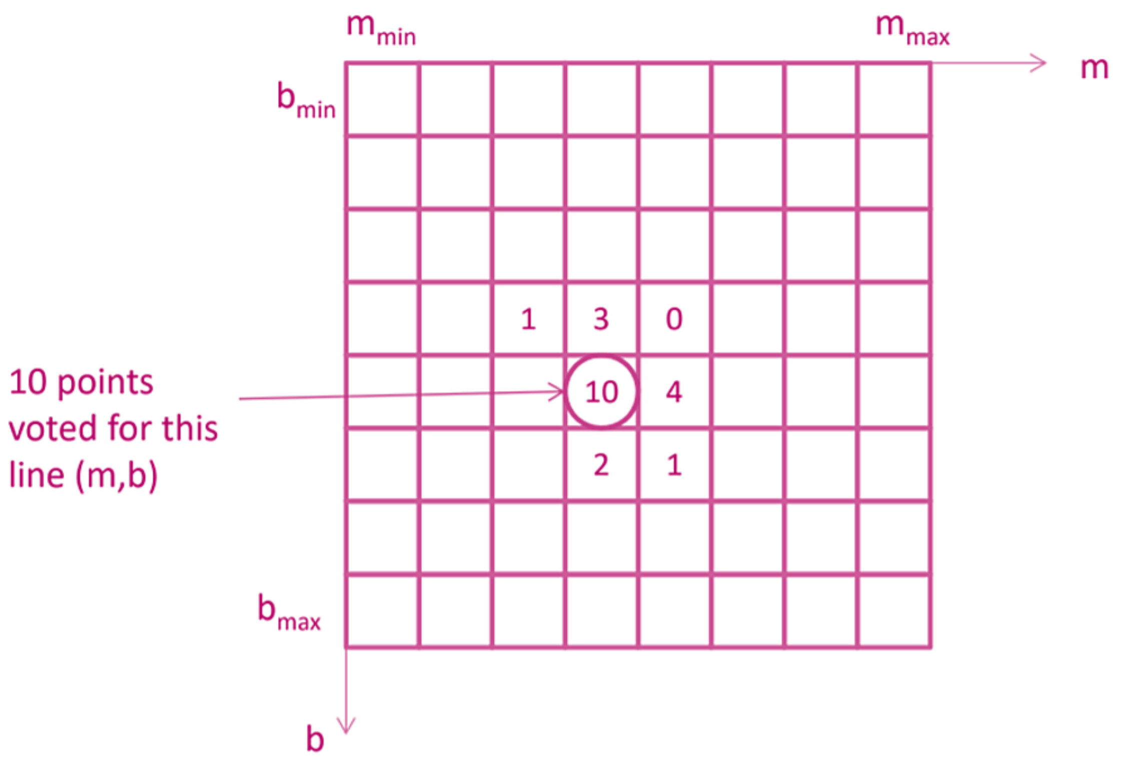

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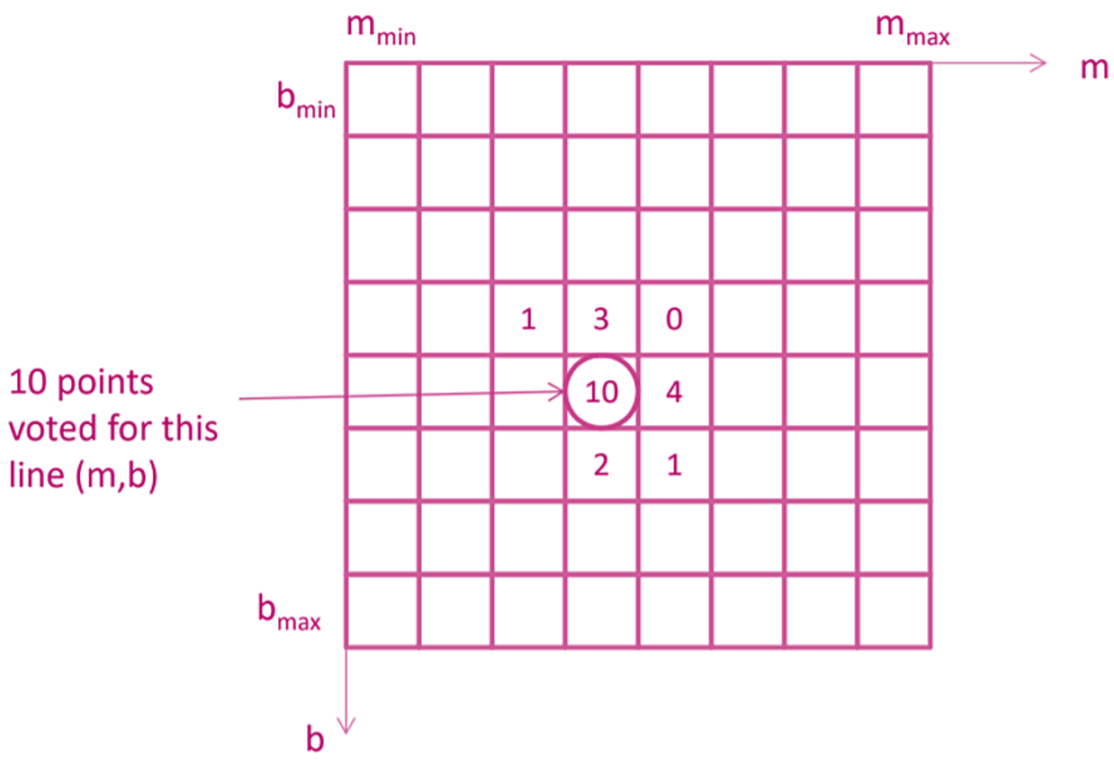

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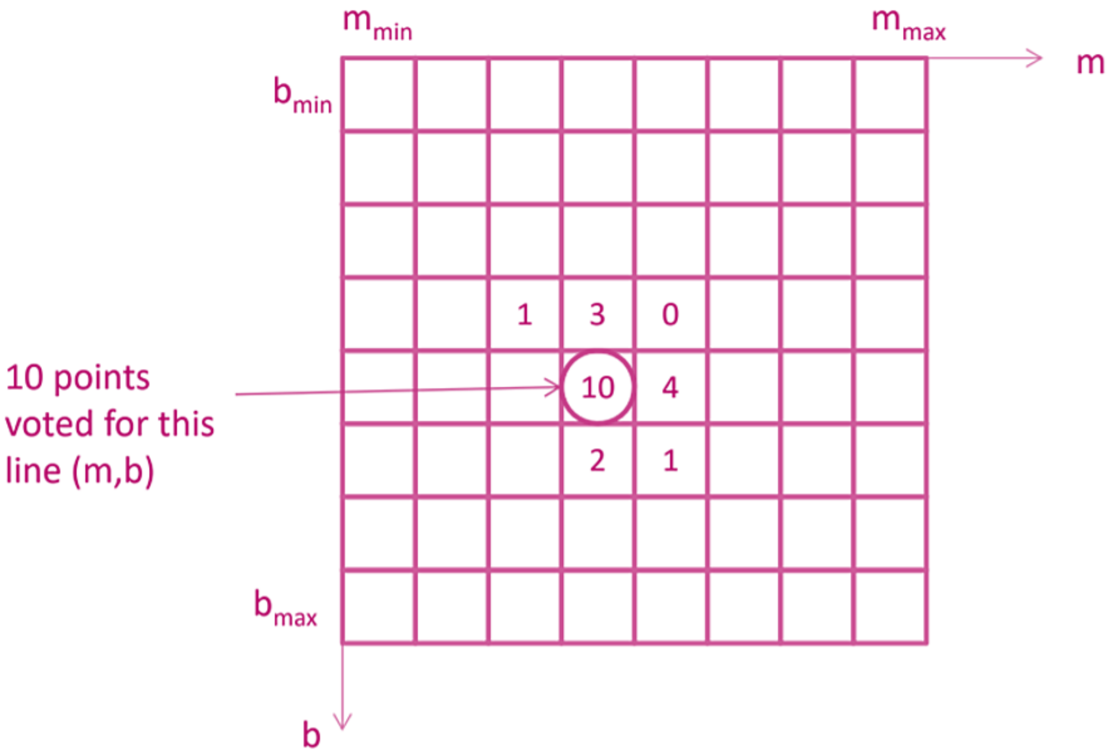
      - m → ∞



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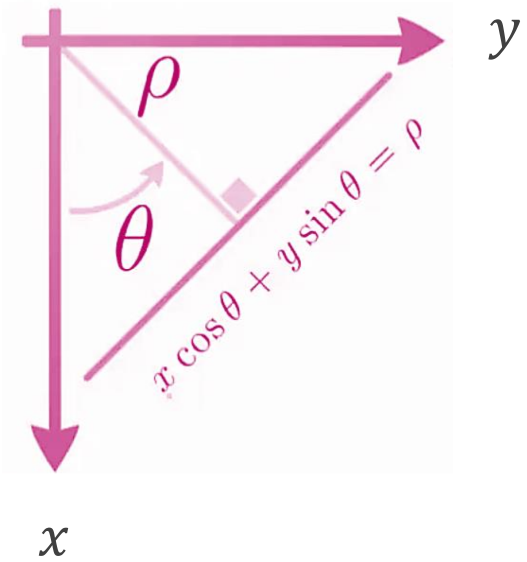

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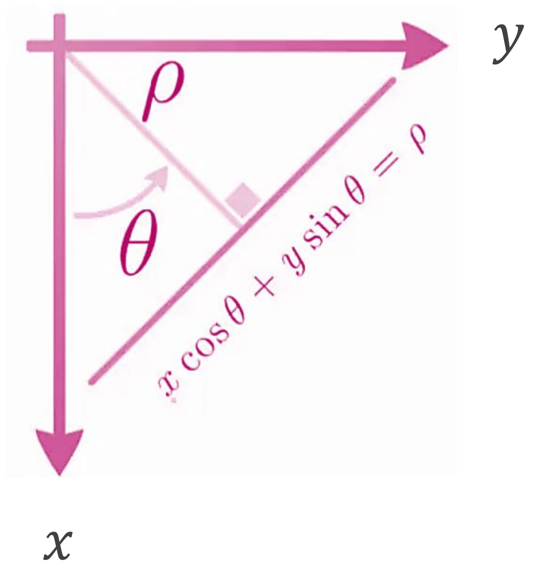
- Vertical lines
  - $\theta = 90^o$



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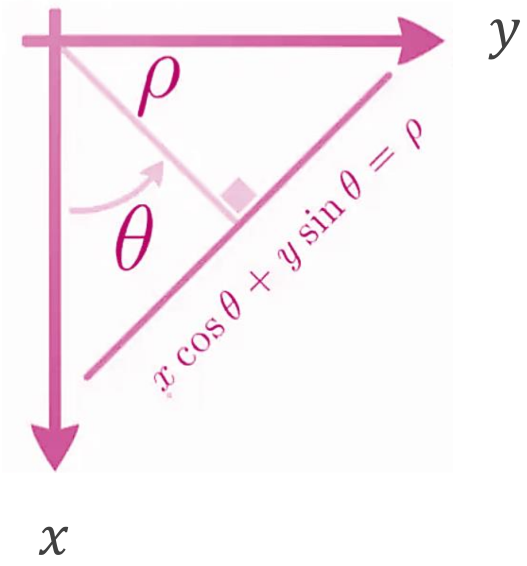
  - $\rho \in [-\mathrm{dmax}, +\mathrm{dmax}]$
  - dmax?



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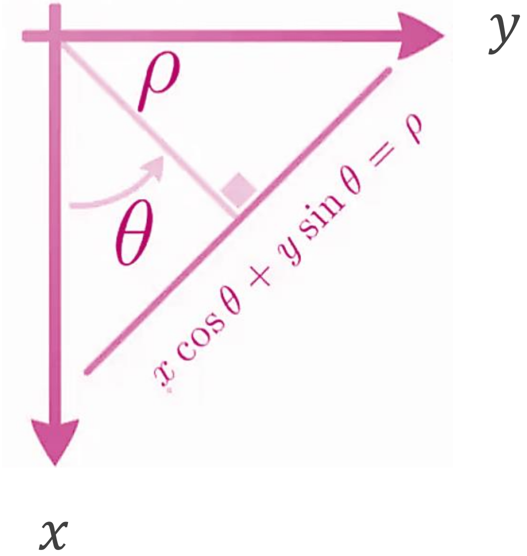

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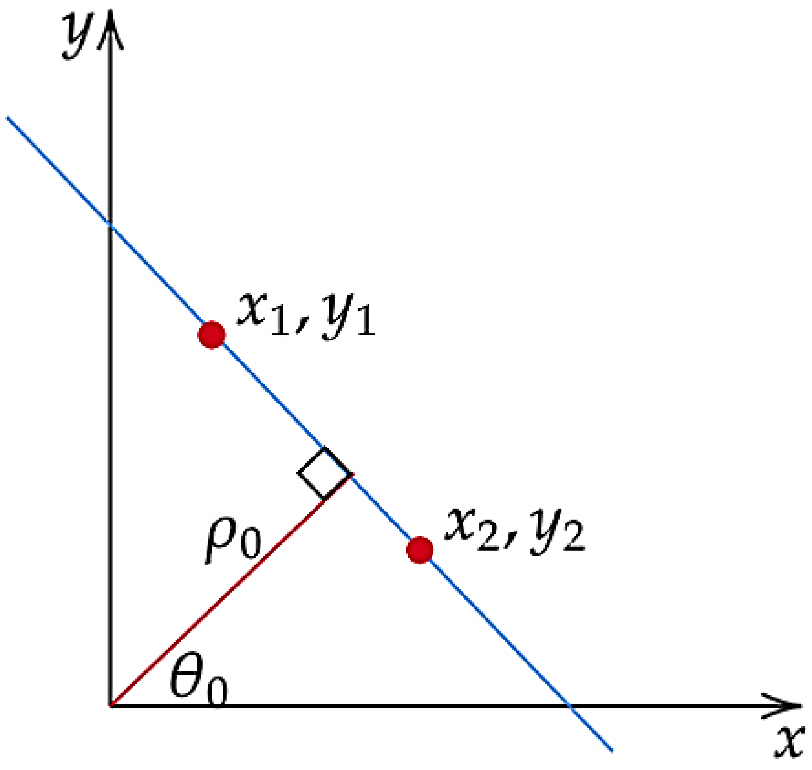

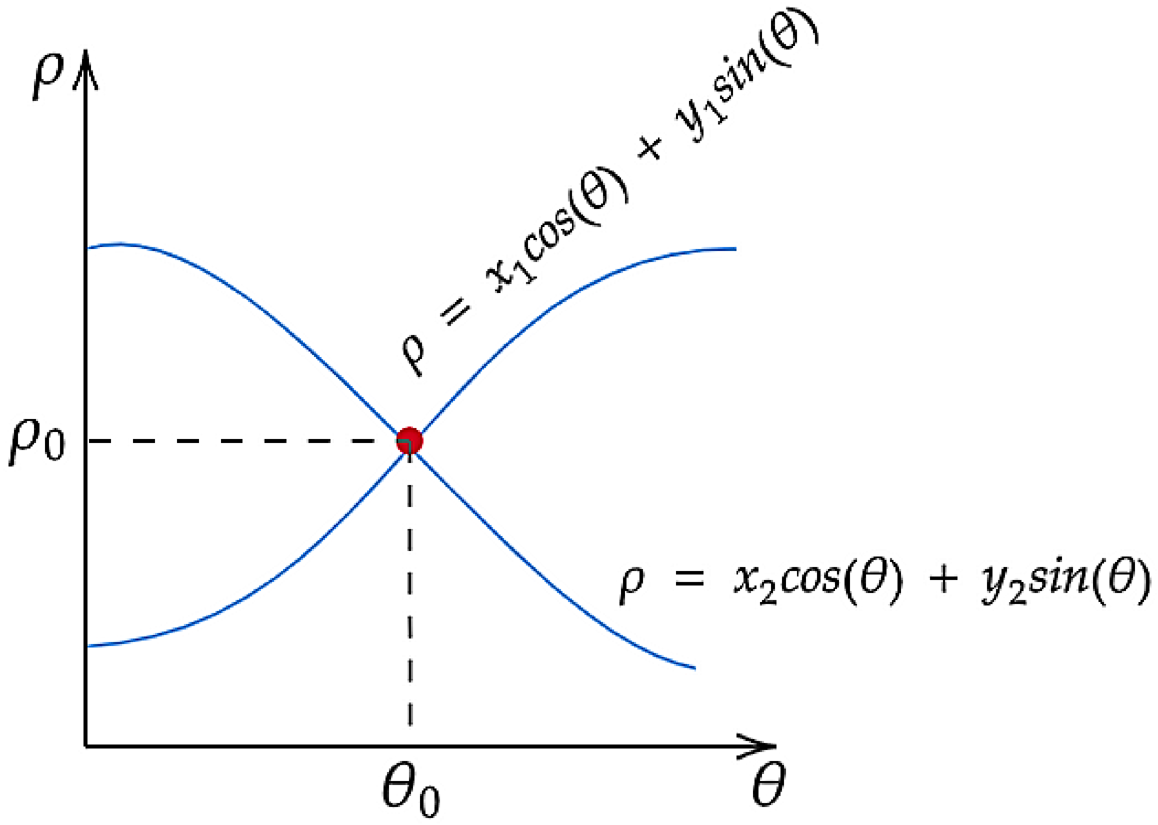

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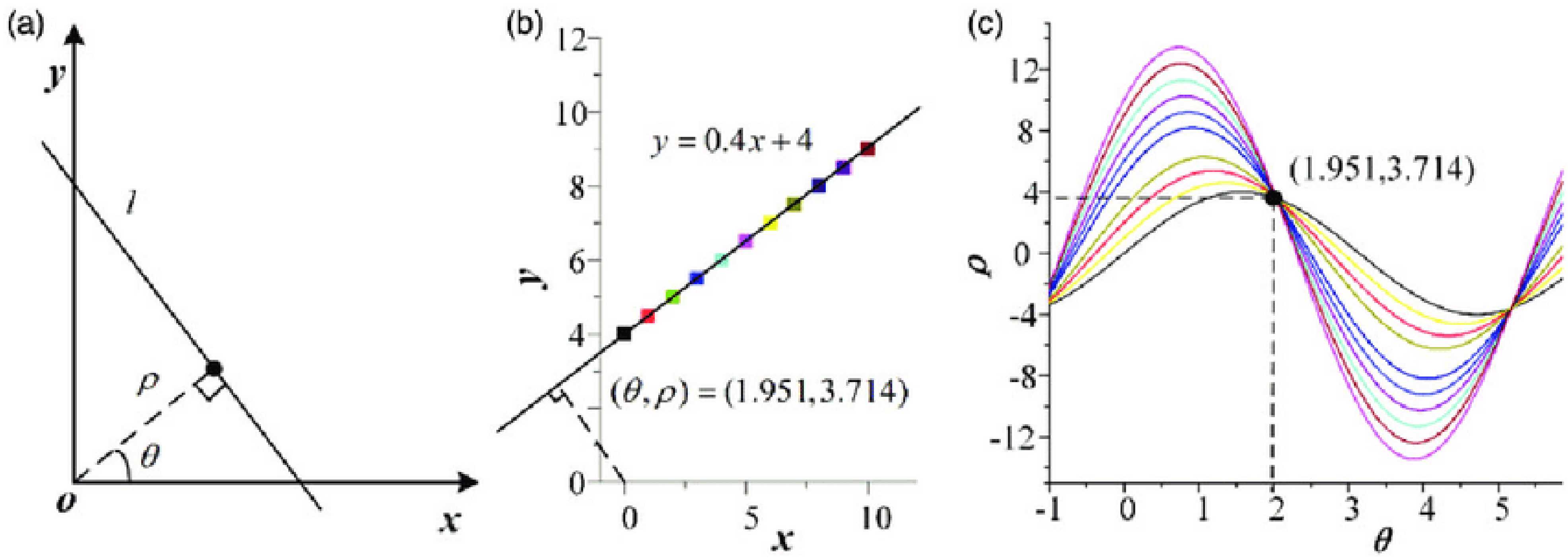

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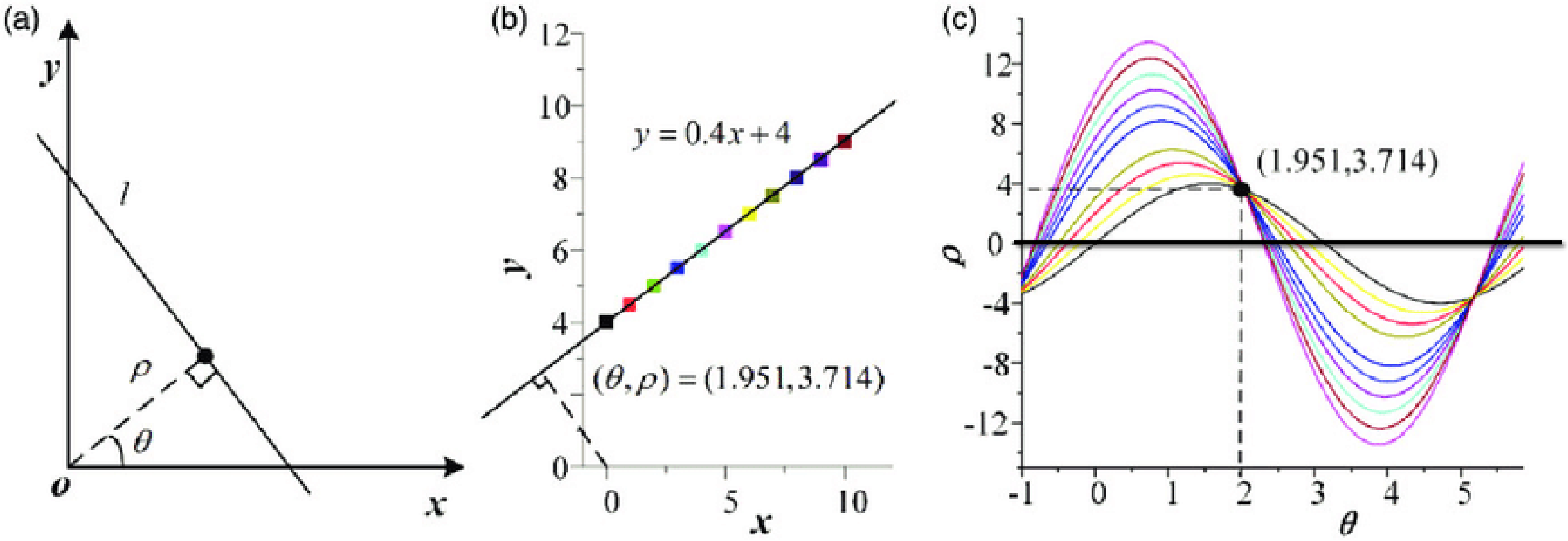

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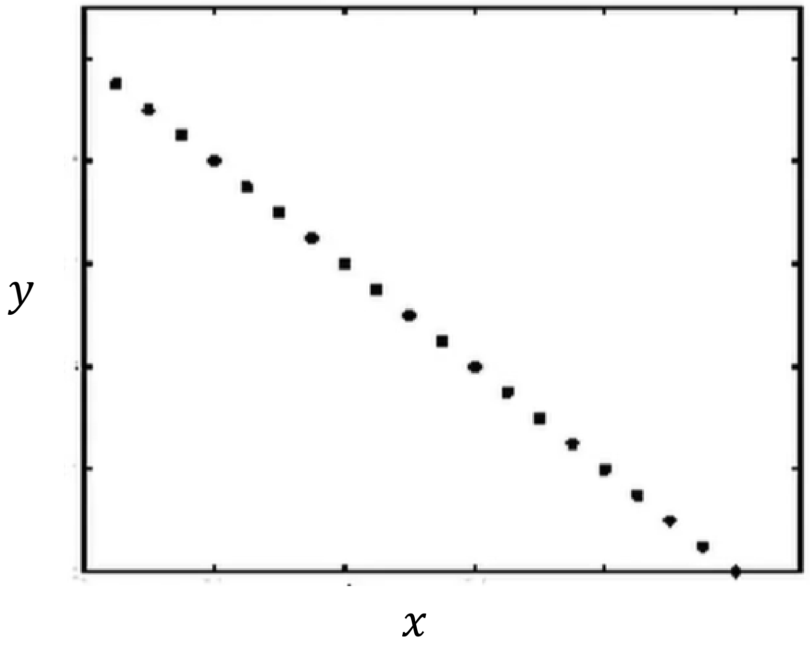

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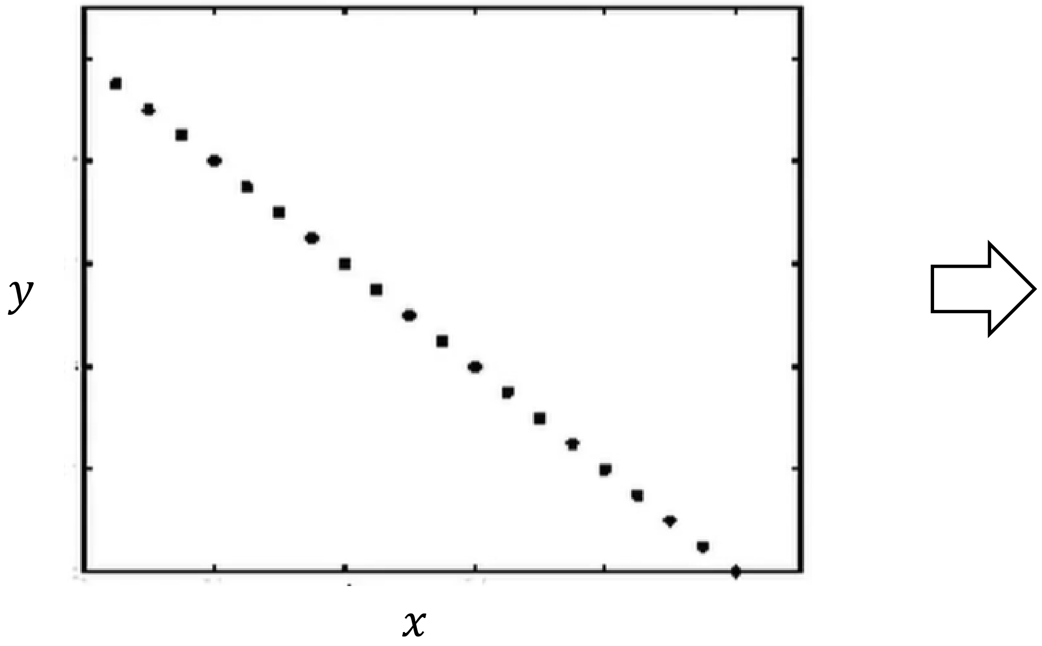

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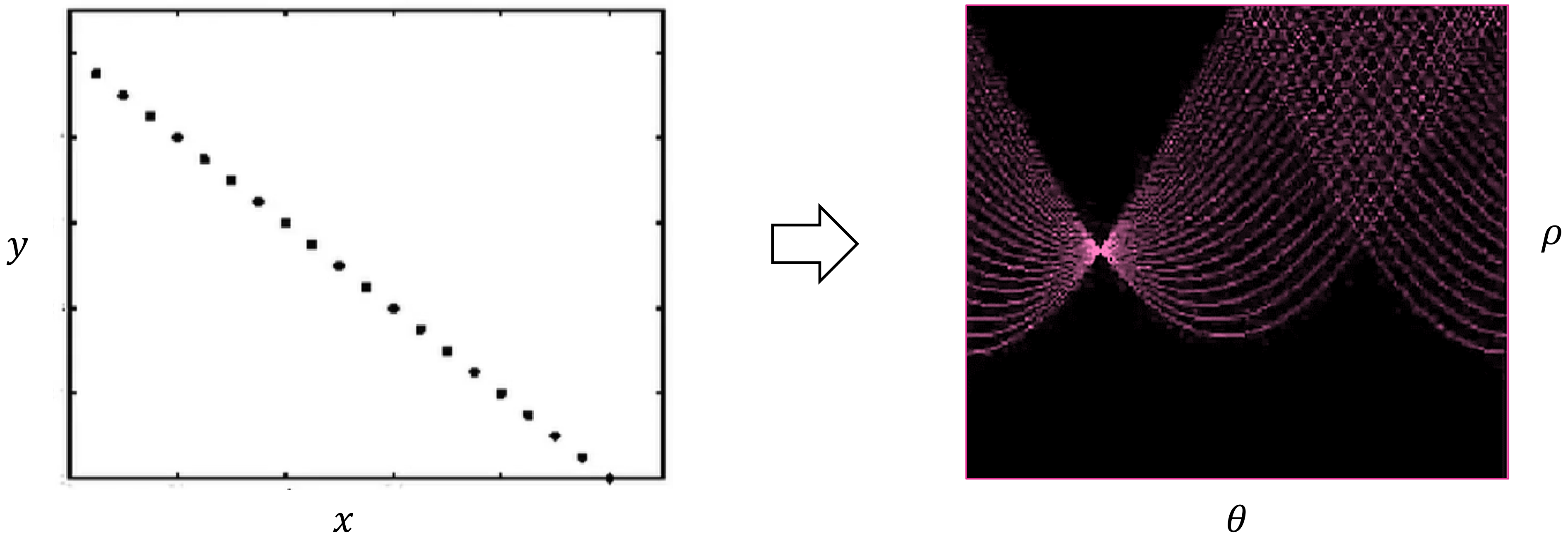

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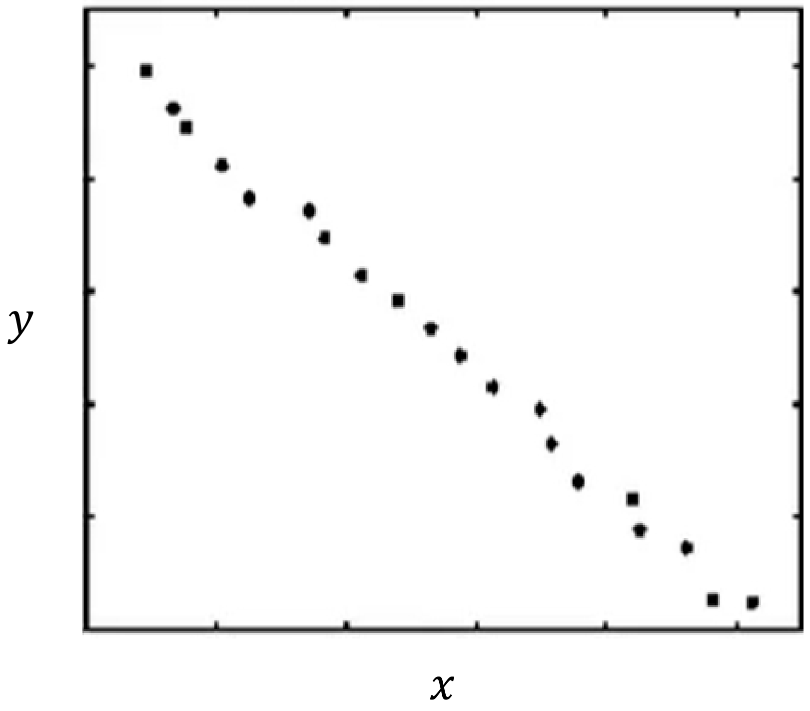
- HT example: with a greater noise



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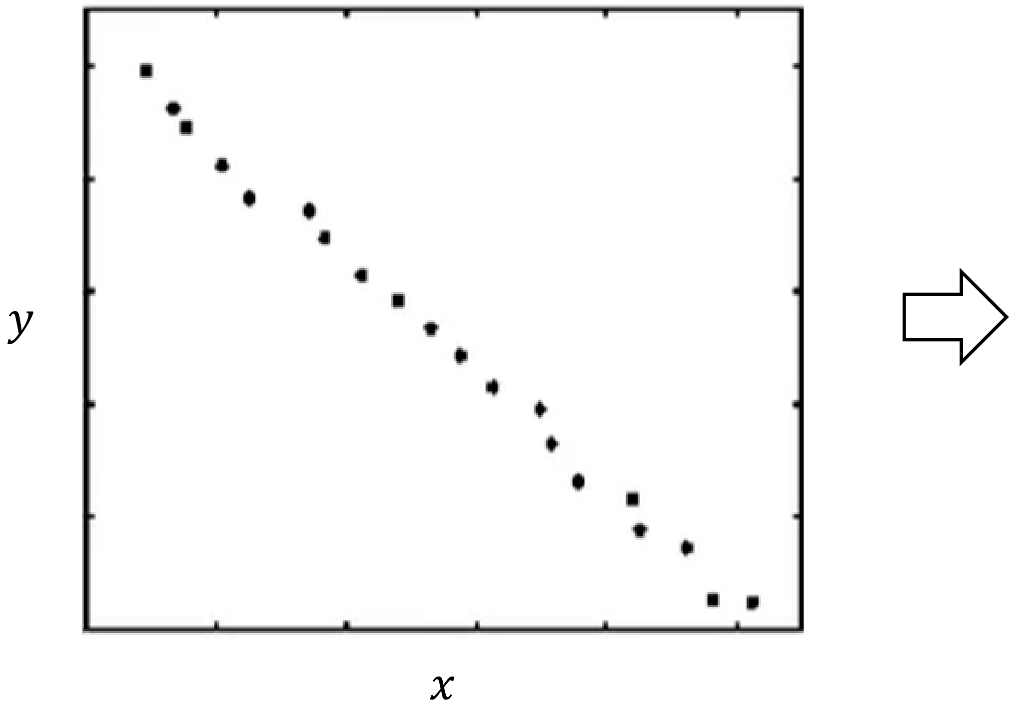

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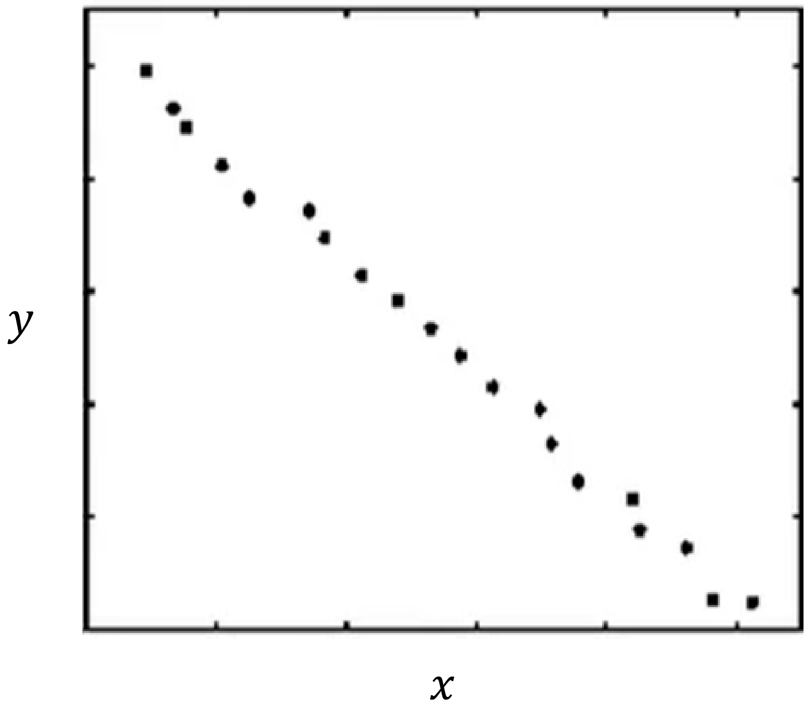

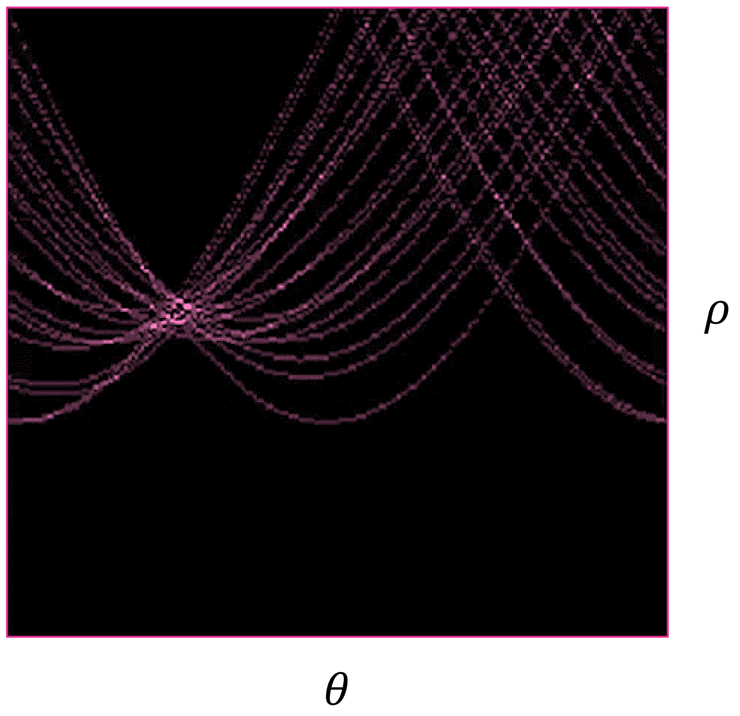

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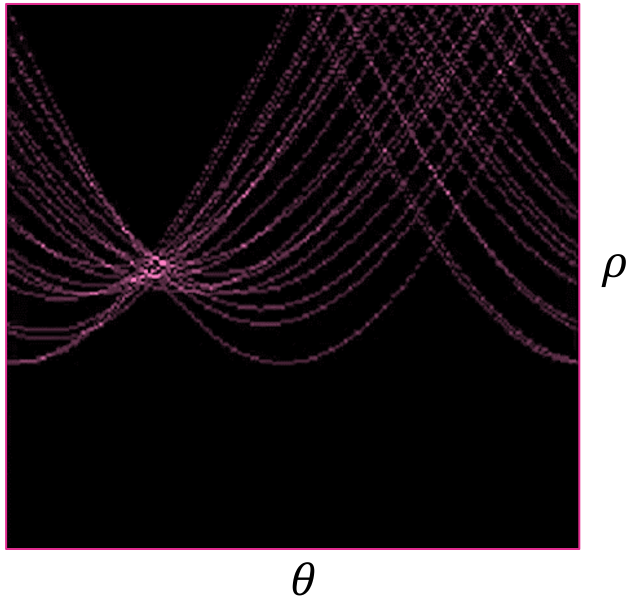
  - tackling the noise



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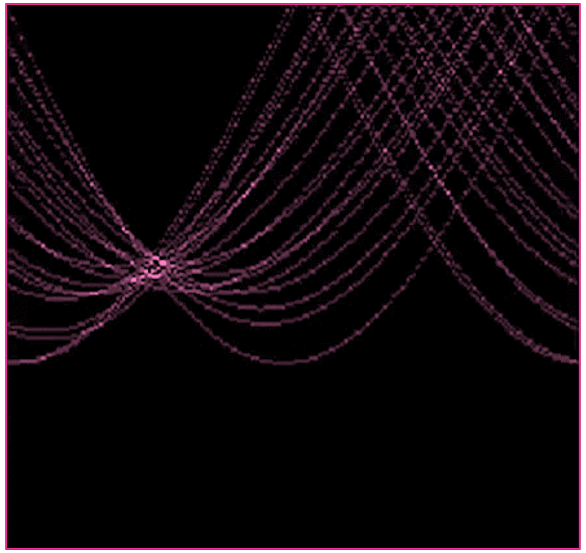
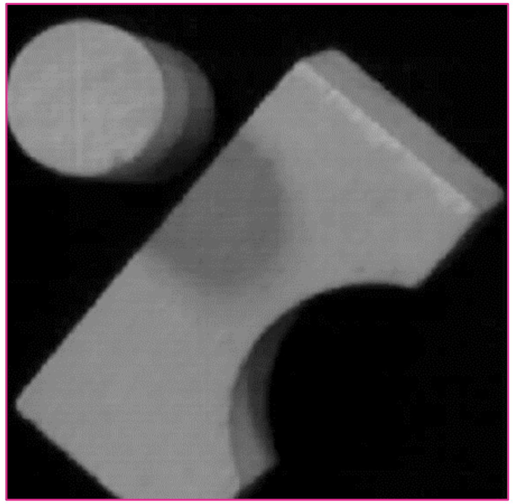
- HT example: with a greater noise
  - tackling the noise



- Image processing in the Hough space
  - smoothing
  - thresholding
  - zoom in the space
  - re-quantize zoomed space
  - redo HT in zoomed space

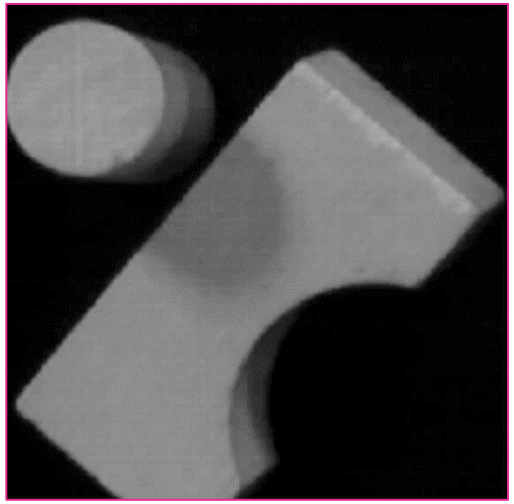
Hough transform

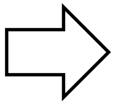
input

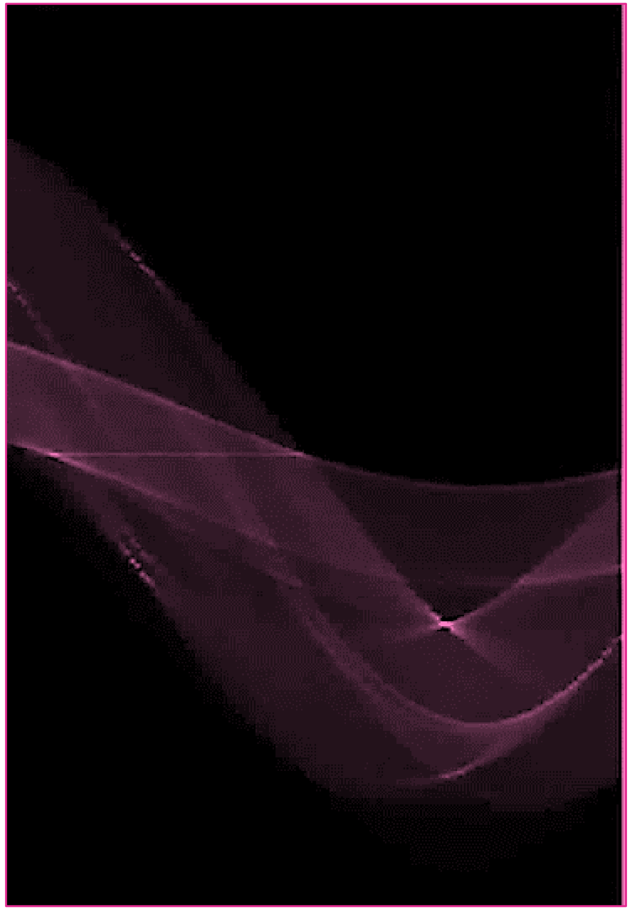


Hough transform

input

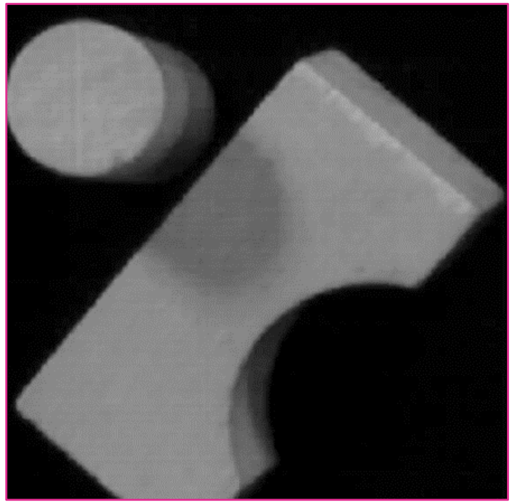




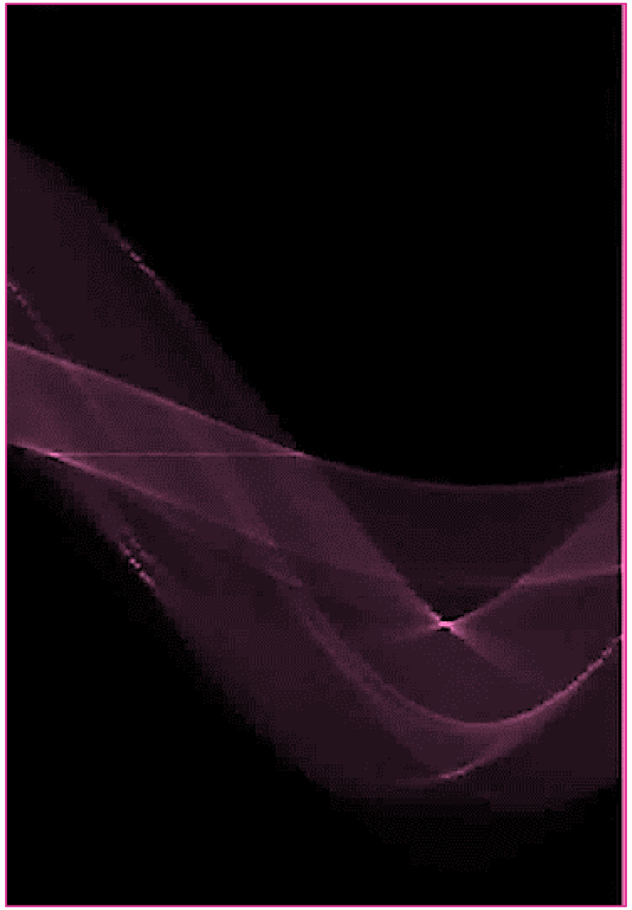


Hough transform

input



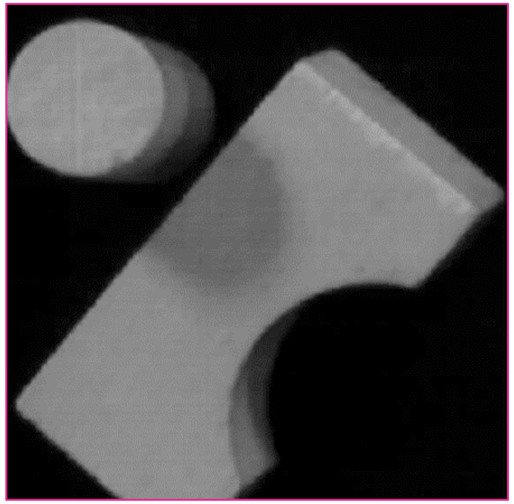
Hough transform

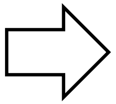


Hough transform

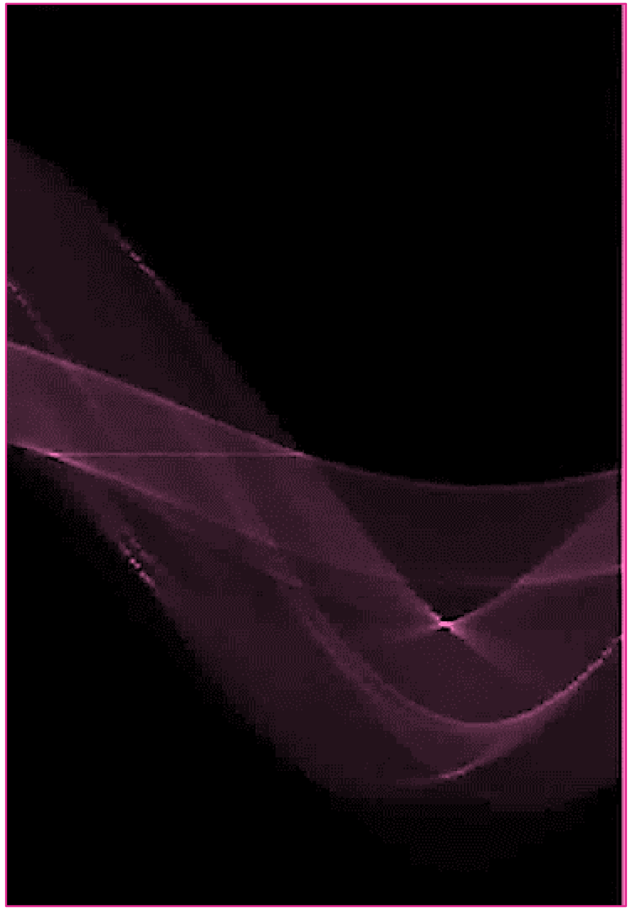
- bright spot in HT corresponds to?

input





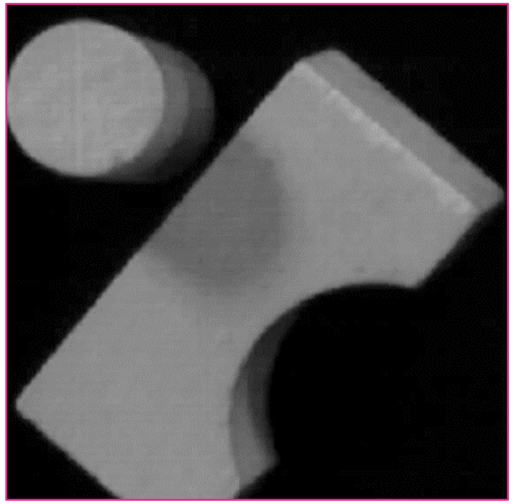
Hough transform



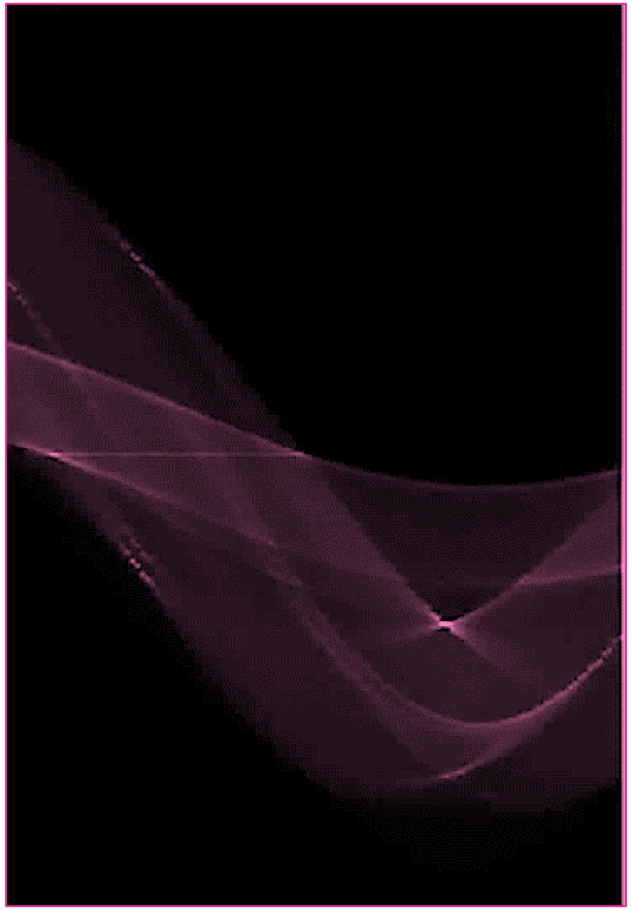
Hough transform

- bright spot in HT corresponds to?
- can circles be detected?

input



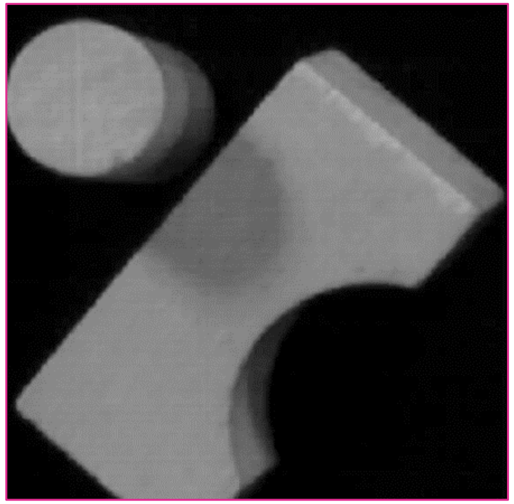
Hough transform



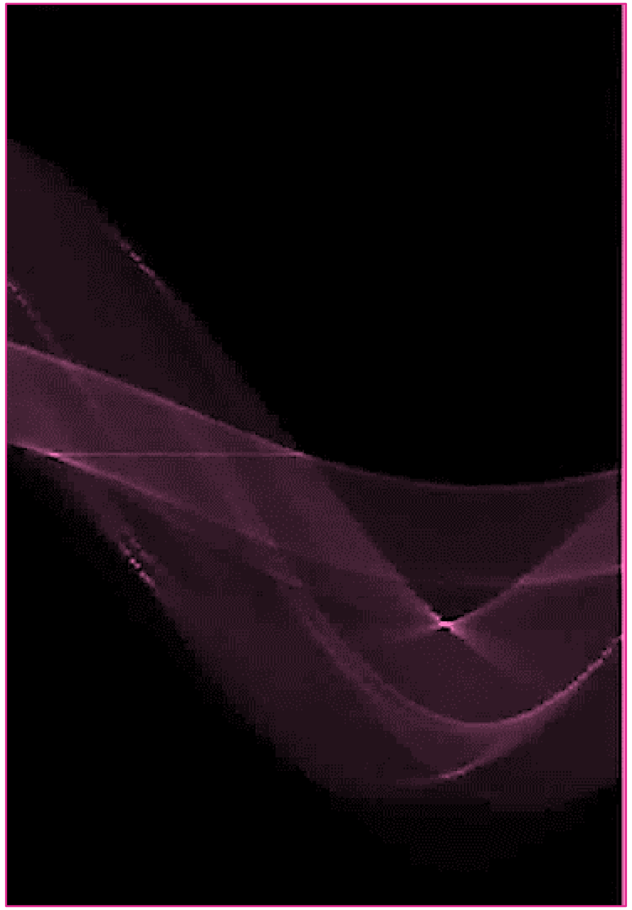
Hough transform

- bright spot in HT corresponds to?
- can circles be detected?
- why does Hough image is not the same size as input?

input



Hough transform



Hough transform

Input

Hough transform

Input

Hough transform

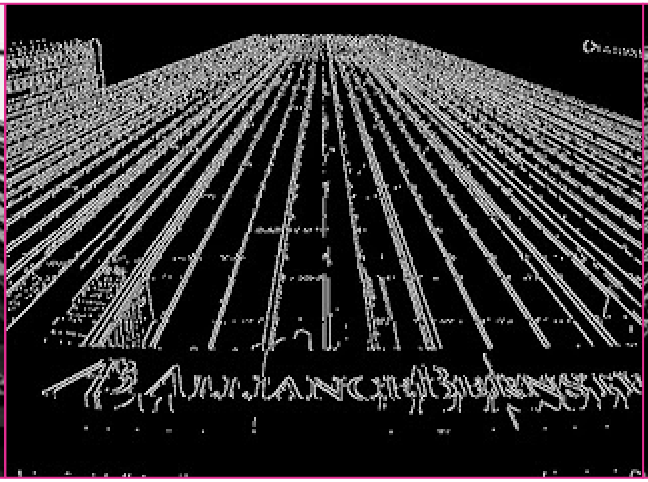
Input

Sobel

Hough transform

Input

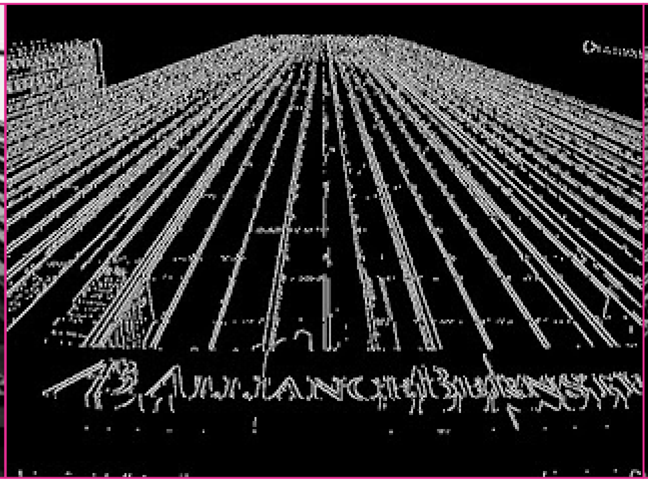
Sobel



Hough transform

Input

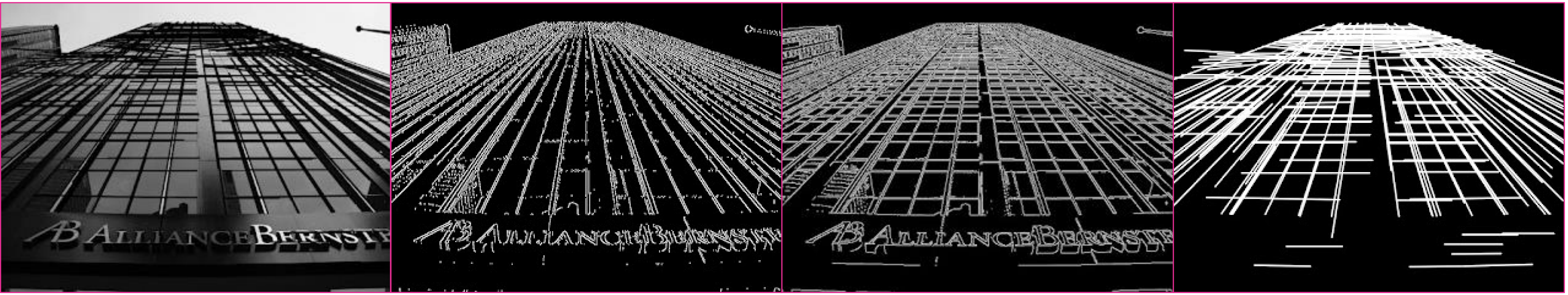
Sobel



Canny

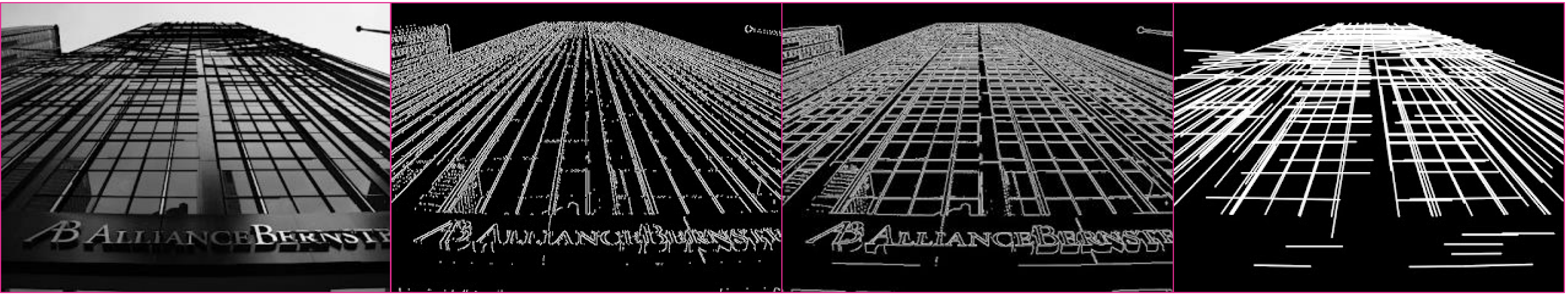
Hough transform

Input Sobel Canny



Hough transform

Input Sobel Canny Hough



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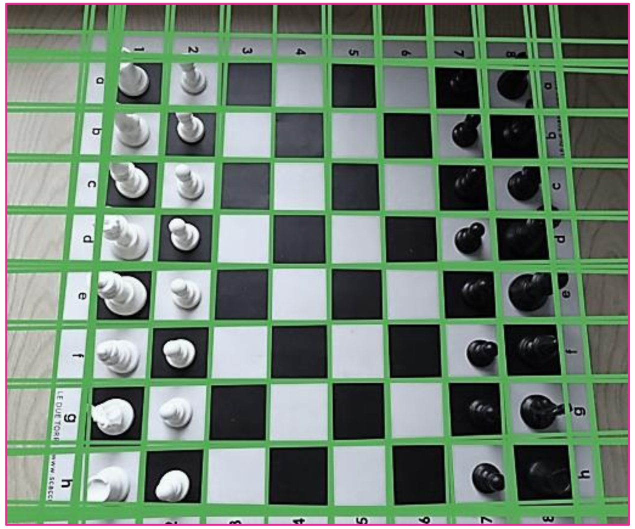

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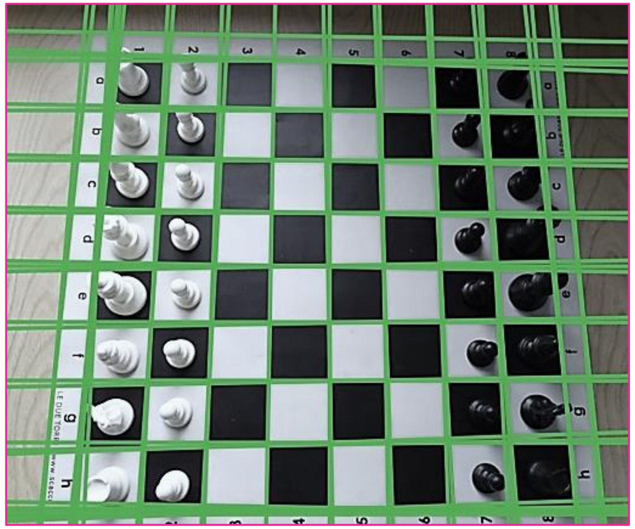

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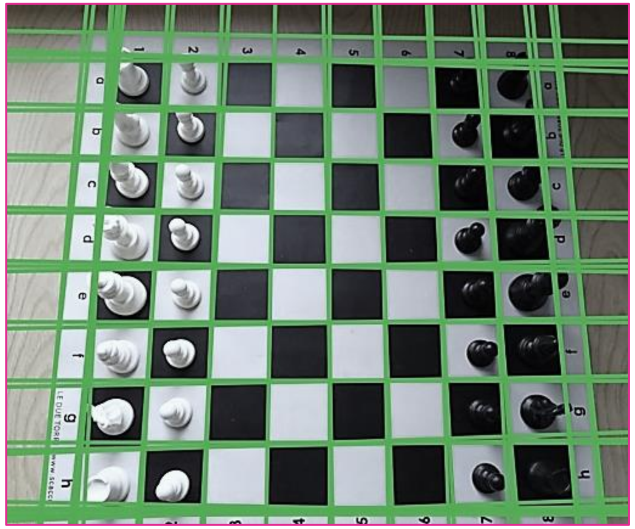

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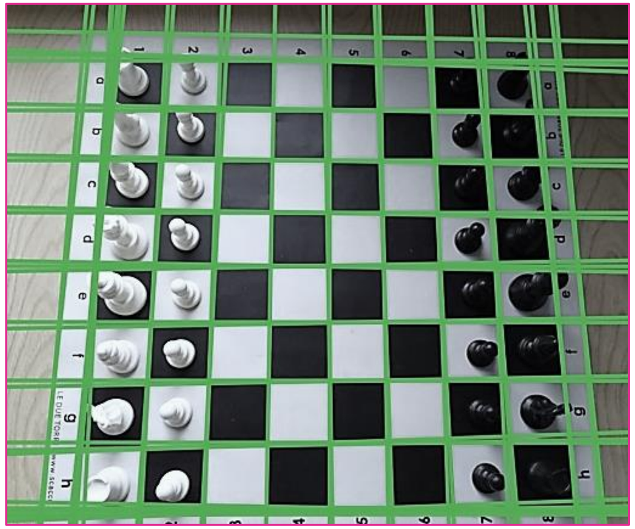
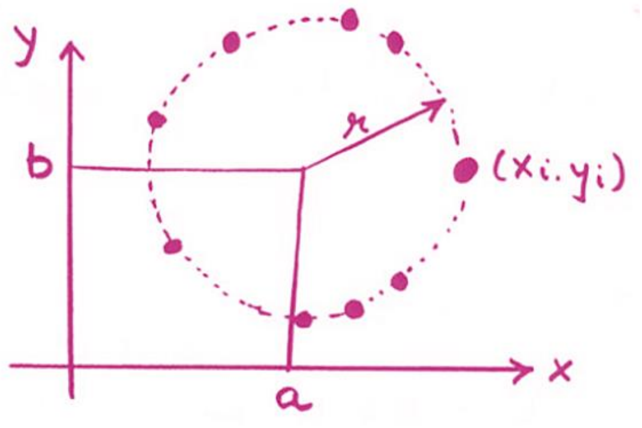


Hough transform

- Equation of circle



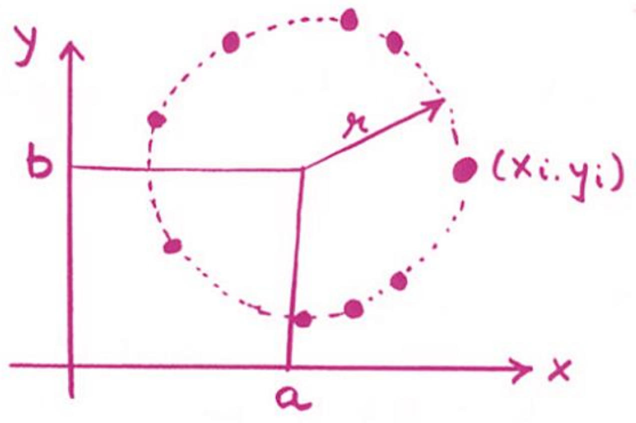
Hough transform

- Equation of circle

$$(x_i - a)^2 + (y_i - b)^2 = r^2$$

If radius is known: (2D Hough Space)

Accumulator Array $A(a,b)$



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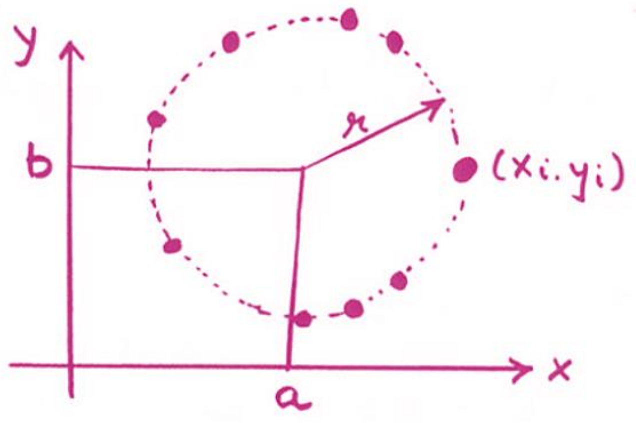

- Approx. object's size are known
  - radius is known

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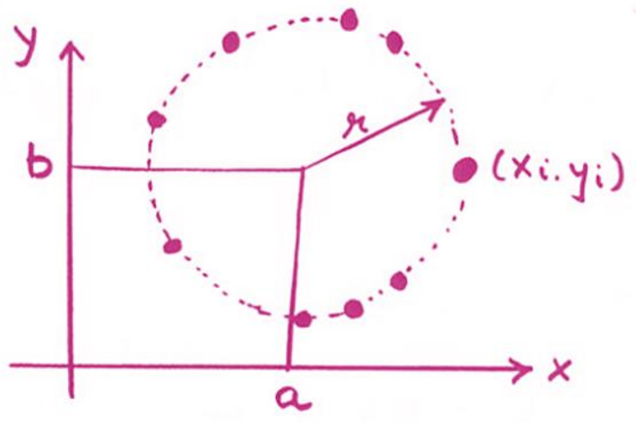

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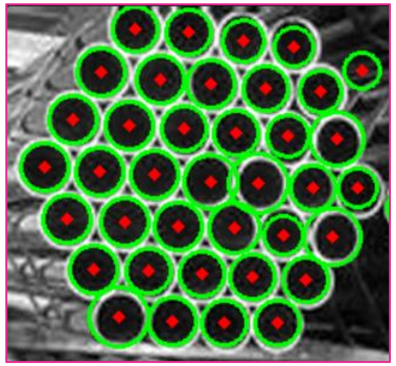
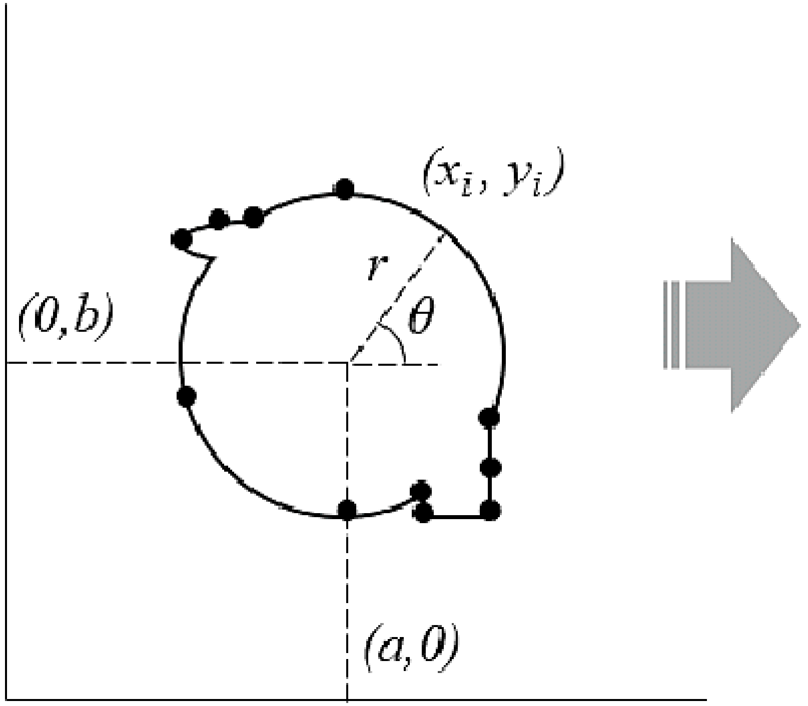
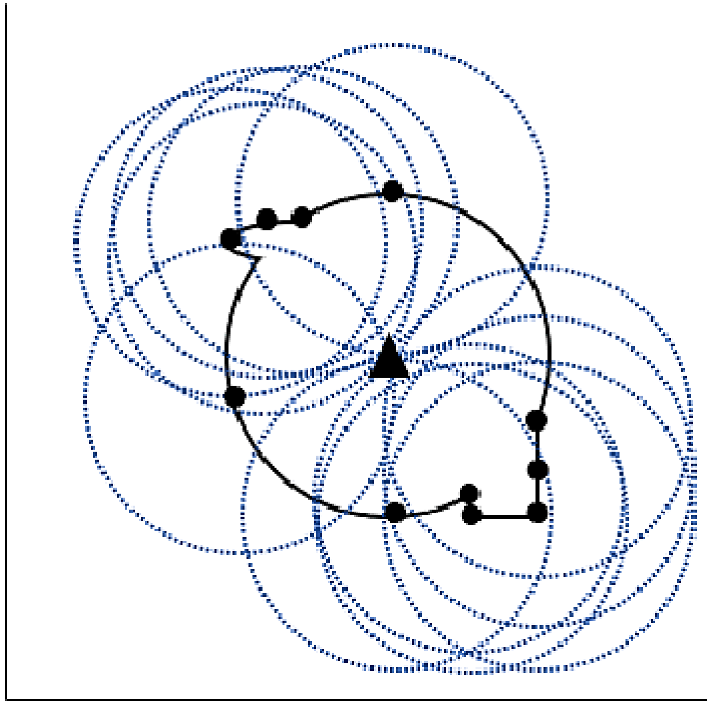


Hough transform



(a) Image space



(b) Parameter space

Hough transform

- If radii are not known, exhaustively search all possibilities

- Pseudocode:

$: A(\quad) = 0$

$: \forall(x, y)$

$\quad : if\ M_T(x, y)$

$\quad\quad : \forall(a, b)$

$\quad\quad\quad : r = sqrt\{\ (x-a)^2 + (y-b)^2\ \}$

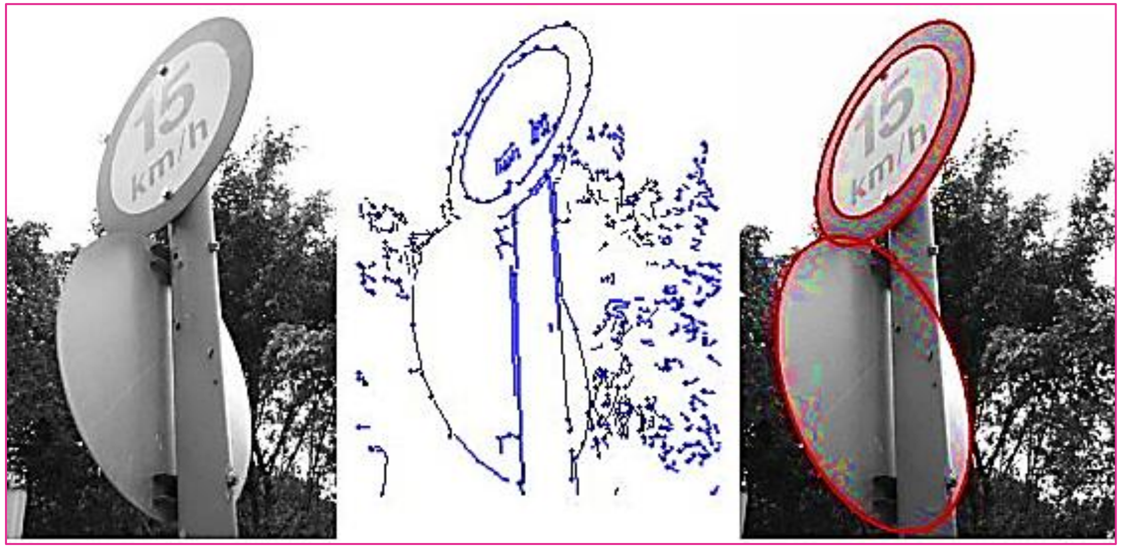
$\quad\quad\quad : A(a, b, r) + +$

$: find\ maximas\ in\ A(\quad)$

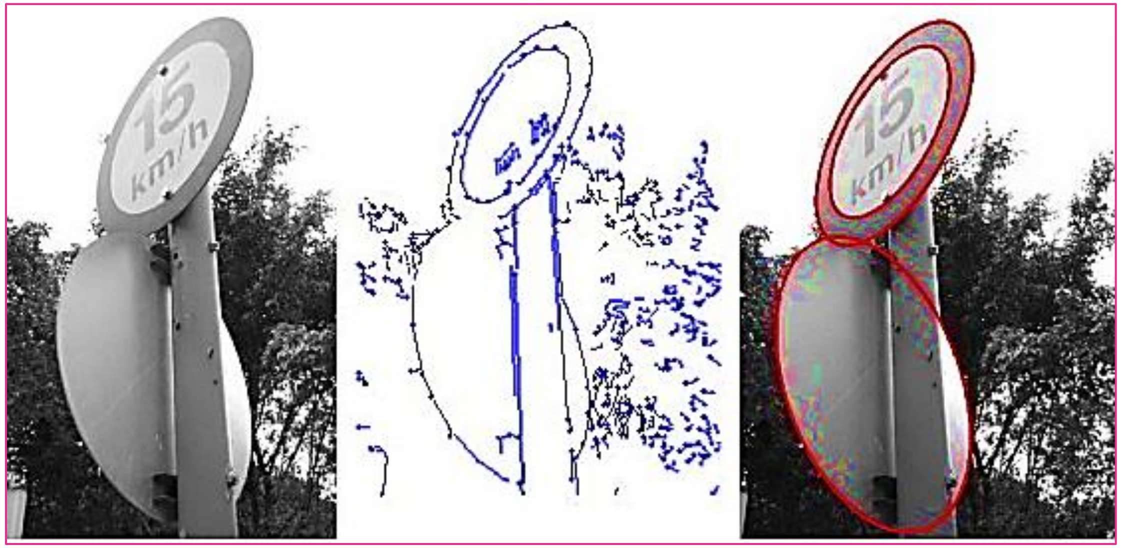
Hough transform

Hough transform

Hough transform

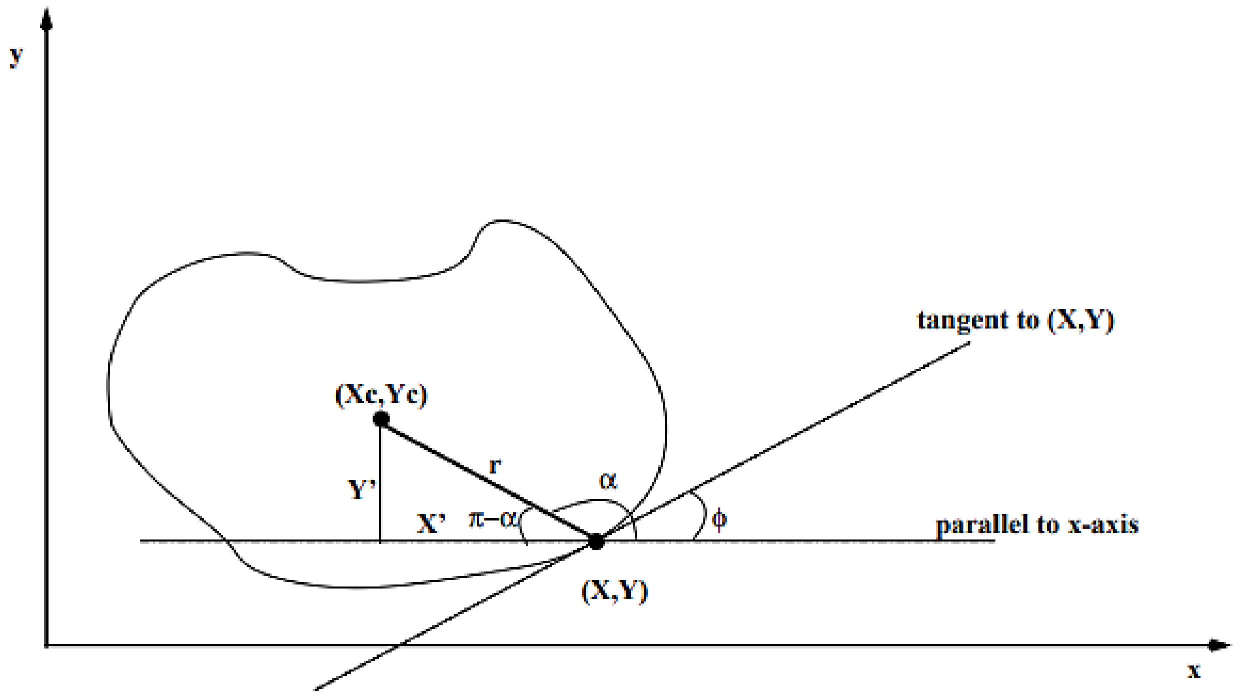


Hough transform



Generalized Hough Transform

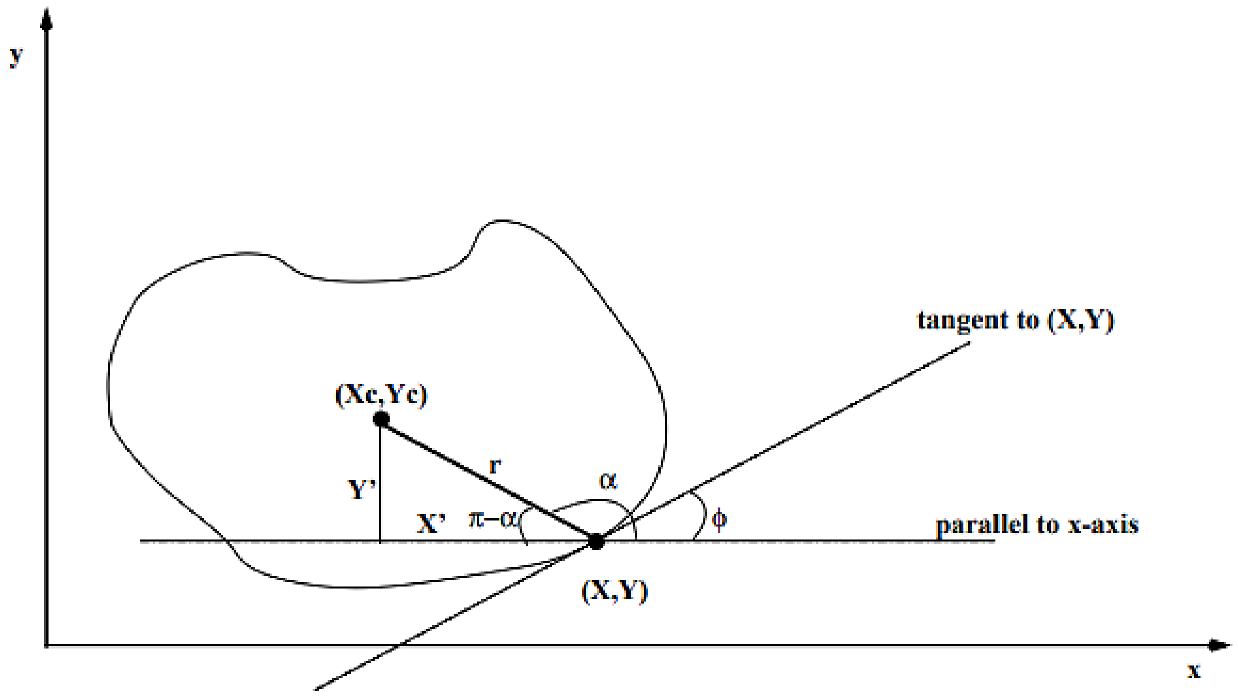
- For any arbitrary shape



Generalized Hough Transform

- For any arbitrary shape

$$
\begin{aligned}
x &= x_c + x' \quad \text{or} \quad x_c = x - x' \\
y &= y_c + y' \quad \text{or} \quad y_c = y - y'
\end{aligned}
$$



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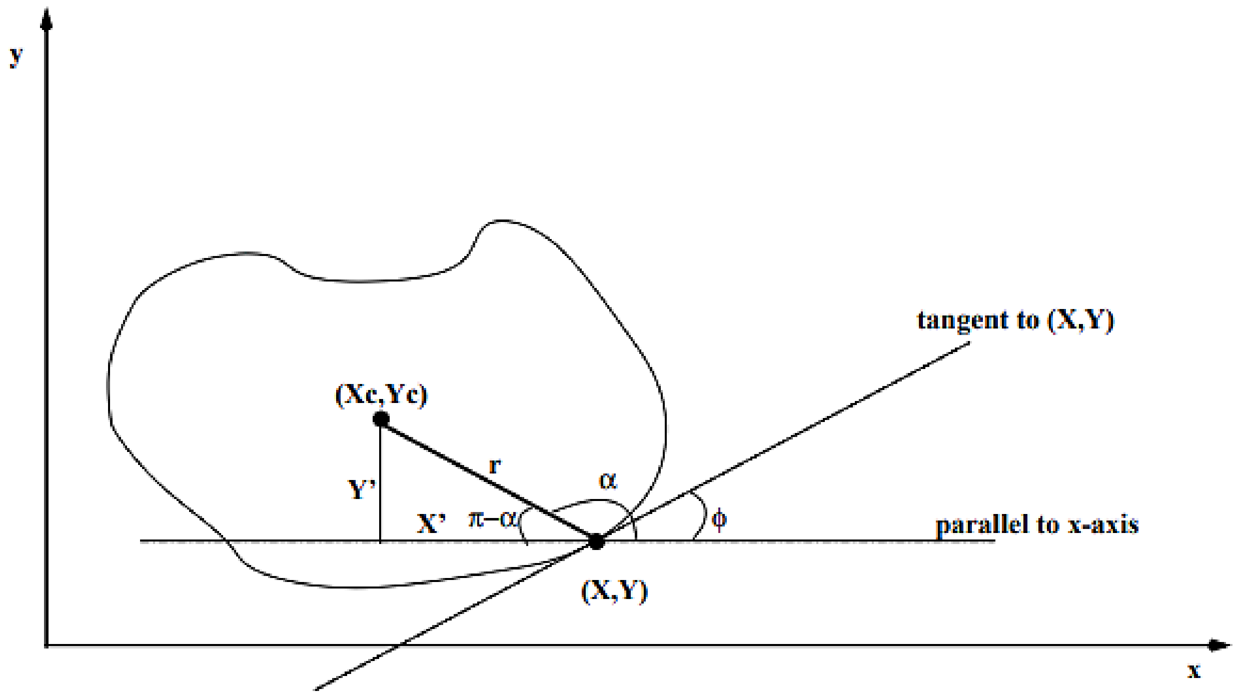
x'



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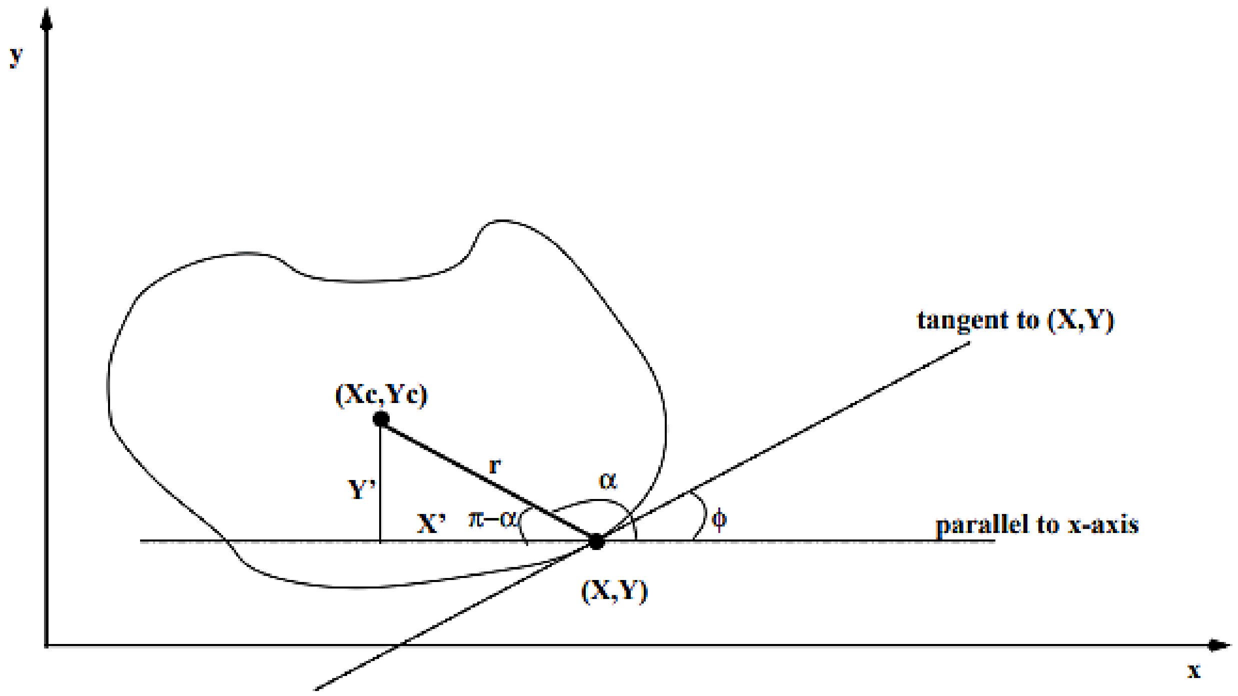
$$x' = rcos(\pi - \alpha) = -rsin(\alpha)$$



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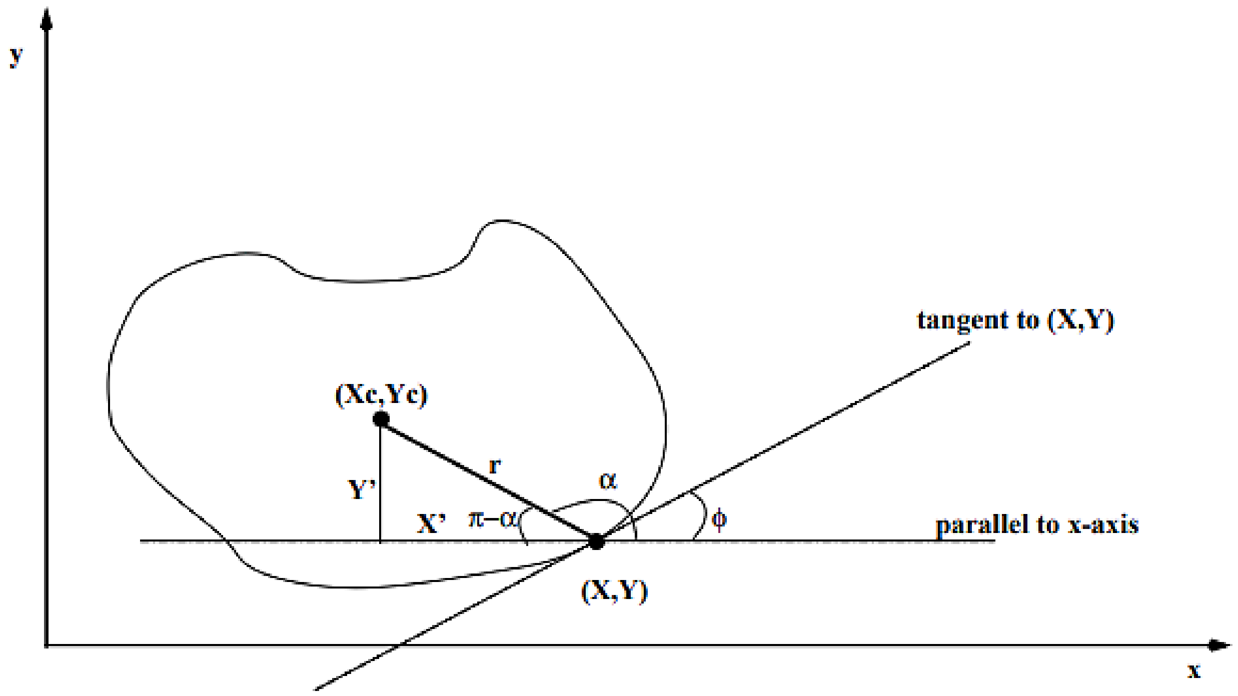
$$y'$$



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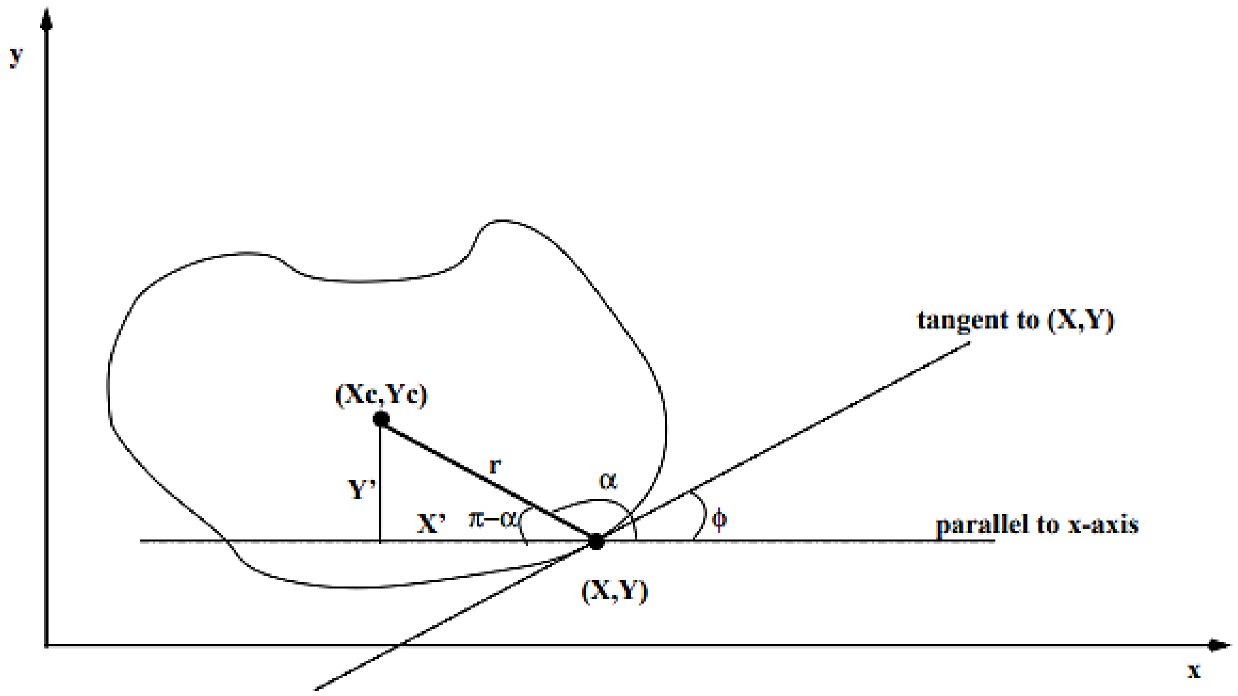

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$$

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\begin{aligned}
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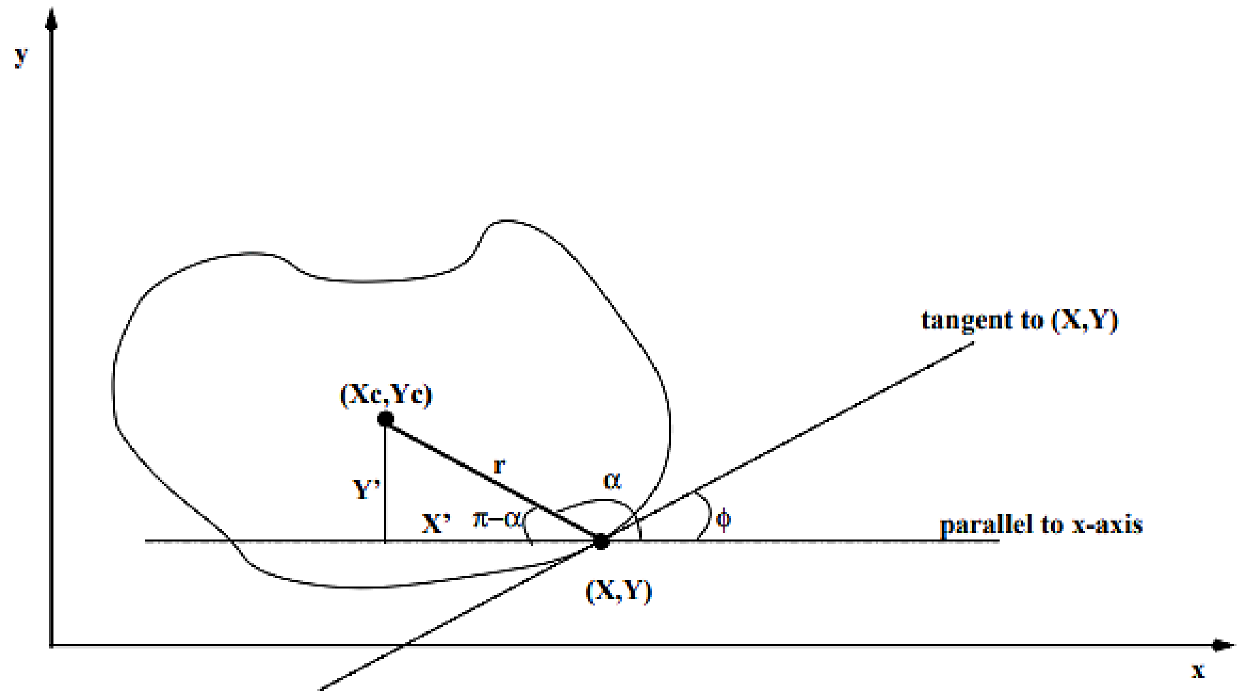

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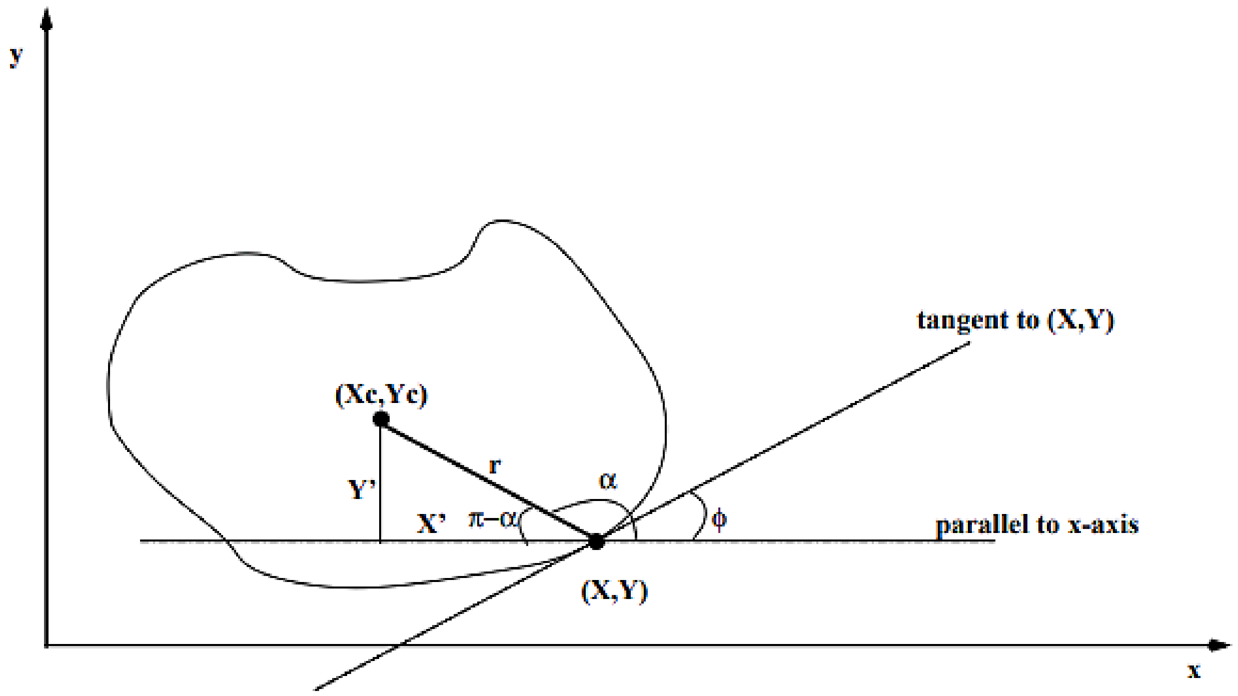

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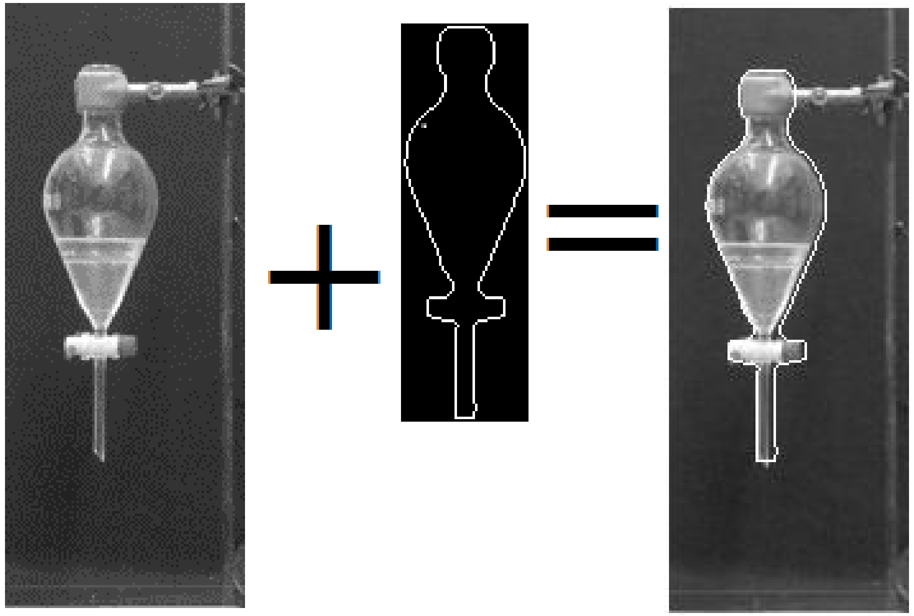
Edge Direction	$\vec{r} = (r, \alpha)$
ϕ_1	$\vec{r}_1^{\,1}, \vec{r}_2^{\,1}, \vec{r}_3^{\,1}$
ϕ_2	$\vec{r}_1^{\,2}, \vec{r}_2^{\,2}$

Generalized Hough Transform

Ref: Mathworks & prof Nayar

Generalized Hough Transform



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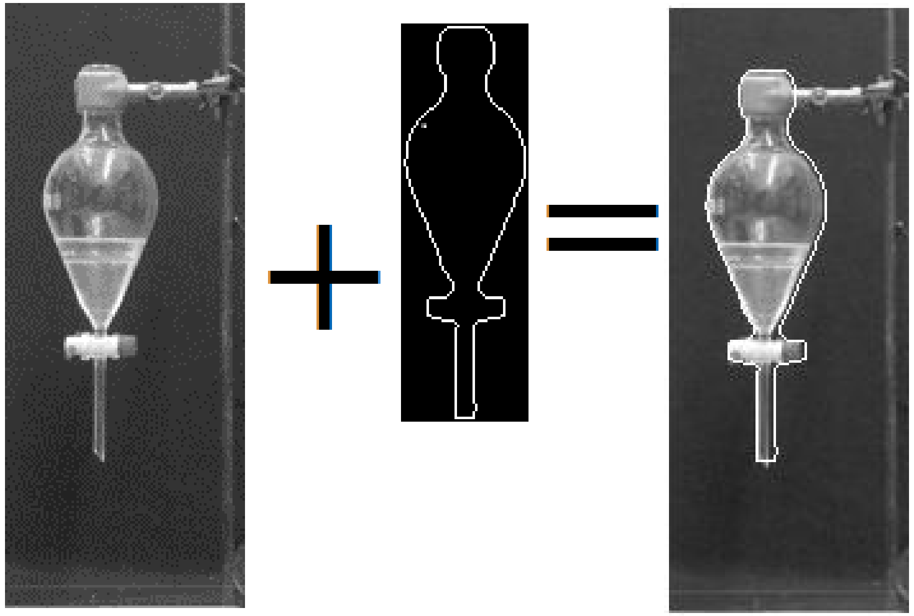
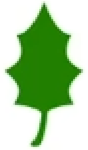
Generalized Hough Transform





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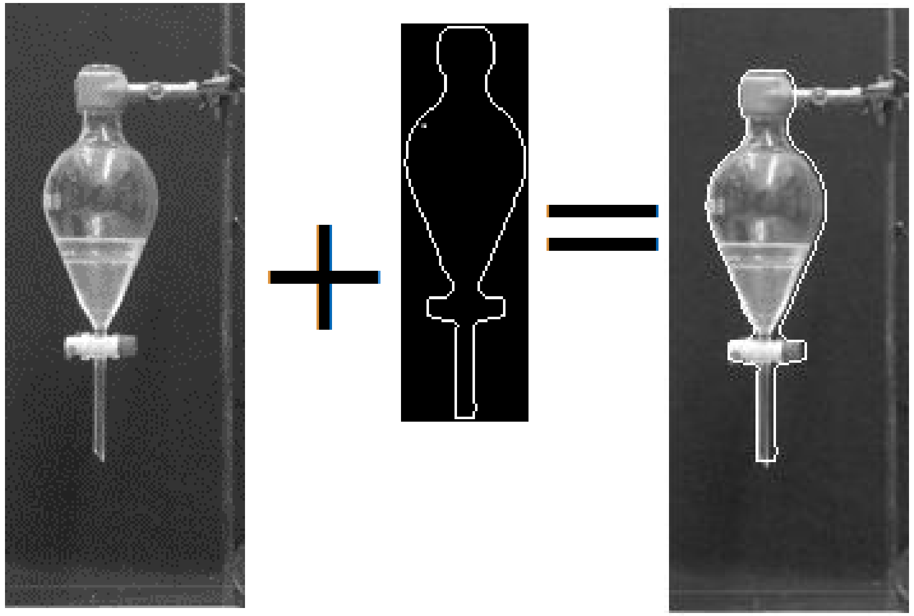

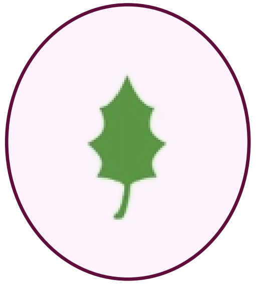

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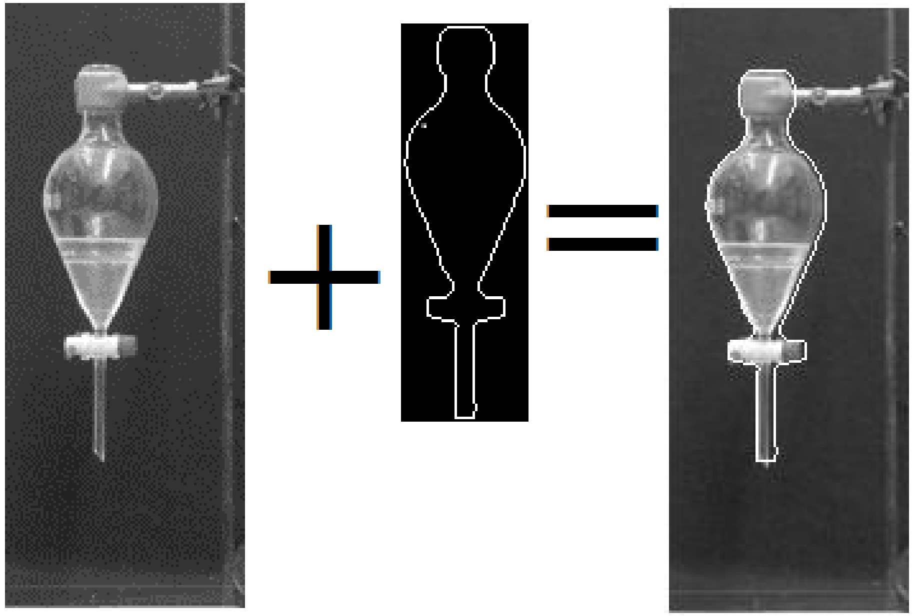

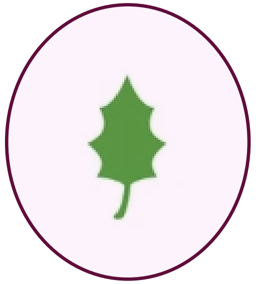

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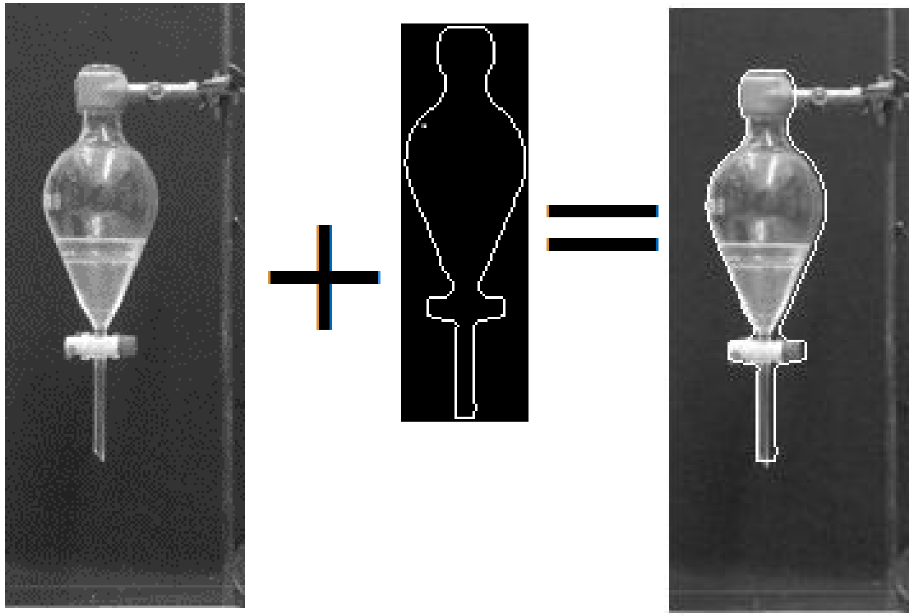

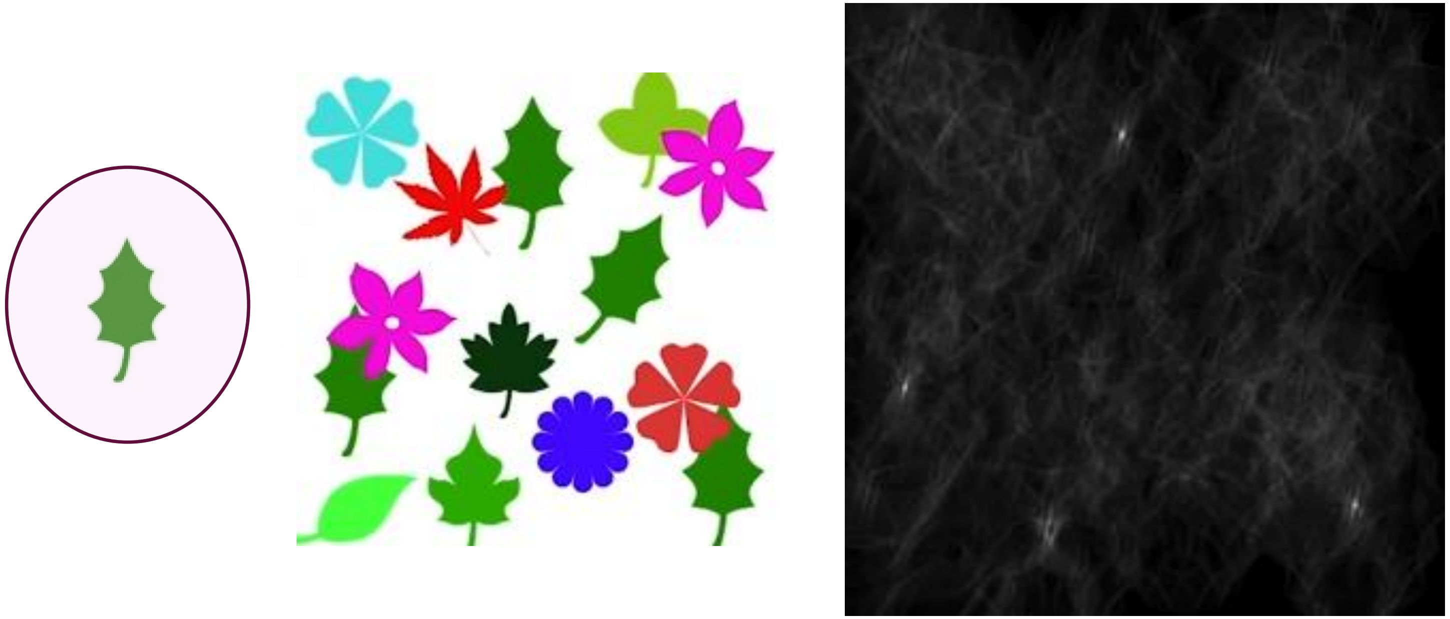

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Hough Transform

- Space complexity

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  - With k quantized bins, it exponentially increases with number of parameters $n \quad \rightarrow \quad k^n$

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Hough Transform

- Space complexity

  - With k quantized bins, it exponentially increases with number of parameters $n \rightarrow k^n$

  - Separate quantization for each dimension can be performed

- Time complexity

  - Voting is linearly proportional to # of edge points
  - Time complexity is constant in # of edge points detected

Hough transform: other shapes

	parameters	
Line	ρ, θ	$x\cos\theta + y\sin\theta = \rho$
Circle	x_0, y_0, ρ	$(x-x_o)^2+(y-y_0)^2=r^2$
Parabola	x_0, y_0, ρ, θ	$(y-y_0)^2=4\rho(x-x_o)$
Ellipse	x_0, y_0, a, b, θ	$(x-x_o)^2/a^2+(y-y_0)^2/b^2=1$

Conclusion

- Hough Transform

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- It's tolerant to noise in some extent and highly tolerant to the missing points in an edge

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  - finer the quantization,
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Hough? →